Analytical and Simulation Method Validation for Flutter Aeroservoelasticity Studies

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ABSTRACT

Aeroservoelasticity studies are interactions between three main disciplines: aerodynamics, aeroelasticity and servo-controls. In the aeroelasticity area, is known that aerodynamic unsteady forces for a range of frequencies and Mach numbers are calculated by use of methods such as Doublet Lattice Method (DLM) in the subsonic regime or by Constant Pressure Methods (CPM) in the supersonic regime. These methods are usually implemented in finite element aeroelasticity software such as Nastran, STARS or any other similar type of software. For aeroelasticity studies, we calculate an aerodynamic unsteady force only for the aircraft elastic modes, while for aeroservoelasticity studies, is necessary to calculate these forces for all aircraft modes: elastic, rigid, elastic and control, not only for its elastic modes. In order to calculate aerodynamic forces for all aircraft modes, we need to consider notions of the flight dynamics theory (Newton's equations) for the aircraft rigid and control modes aerodynamic forces calculations, while the methods implemented in finite element aeroelasticity software will be considered for elastic aerodynamic forces calculations. Therefore, the unsteady aerodynamic forces corresponding to all aircraft modes will be calculated with two different methods: numerical and analytical.

1.0 INTRODUCTION

Aeroservoelasticity studies are very important in the aircraft industry. After an extensive bibliographical research in the field, we did not find a well-documented formulation for rigid and control mode aerodynamic forces for aeroservoelasticity; therefore we have formulated and validated a novel method in this paper.

Finite element aeroelasticity software such as STARS[1] or Nastran[2] does not accurately calculate the rigid and control mode aerodynamic forces, but they do calculate the elastic mode aerodynamic forces. Our new formulation will calculate and validate the aerodynamic rigid and control modes forces on an F/A-18 aircraft by use of the Doublet Lattice Method (subsonic regime) and the Constant Pressure Method (supersonic regime). Thus, rigid and control mode forces calculated with our new formulation will replace the rigid and control modes calculated with finite element-based aeroelasticity software.

The F/A-18 aircraft structure is modelled[3] by finite element methods, and 44 frequencies and mode shapes are calculated for this aircraft, which are divided into the following three groups: 6 rigid modes (3 symmetric and 3 anti-symmetric), 28 elastic modes (14 symmetric and 14 anti-symmetric) and 10 control modes (5 symmetric and 5 anti-symmetric)

The Doublet Lattice Method, DLM, (in subsonic regime) or the Constant Pressure Method, CPM, (in supersonic regime) are used to calculate the unsteady aerodynamic forces Q(k, $Mach$) for various Mach numbers and reduced frequencies. The aerodynamic forces correspond to the following modes: 6 rigid modes (3 modes in translation and 3 modes in rotation), 10 control modes (5 modes for longitudinal and 5 modes for lateral motions) and e represent the 28 elastic modes (14 modes in longitudinal motion and 14 modes in lateral motion).
The aerodynamic forces for the rigid-to-rigid mode interactions Q_{rr} (dimensions 6 * 6) and for the rigid-to-control mode interactions Q_{rc} (dimensions 6 * 10), calculated with finite element software Nastran, will be replaced by Q_{rr} and Q_{rc} values calculated with the two schemes, analytical and numerical, presented in this paper.

2.0 ANALYTICAL VERSUS NUMERICAL FORMULATIONS

The analytical formulation is presented in Sections 1 to 3, and the numerical formulation is presented in Section 4. Details of the first simulation scheme with stability derivatives in the wind system of coordinates are explained in the first section, while Section 2 presents the first scheme with state variables introduced in the aircraft closed loop. Section 3 presents the analytical formulations for the aerodynamic forces for rigid-to-rigid and rigid-to-control interactions mode calculations.

2.1 First simulation scheme with stability derivatives in the wind system of coordinates

The detailed first scheme is shown in Figure 1:

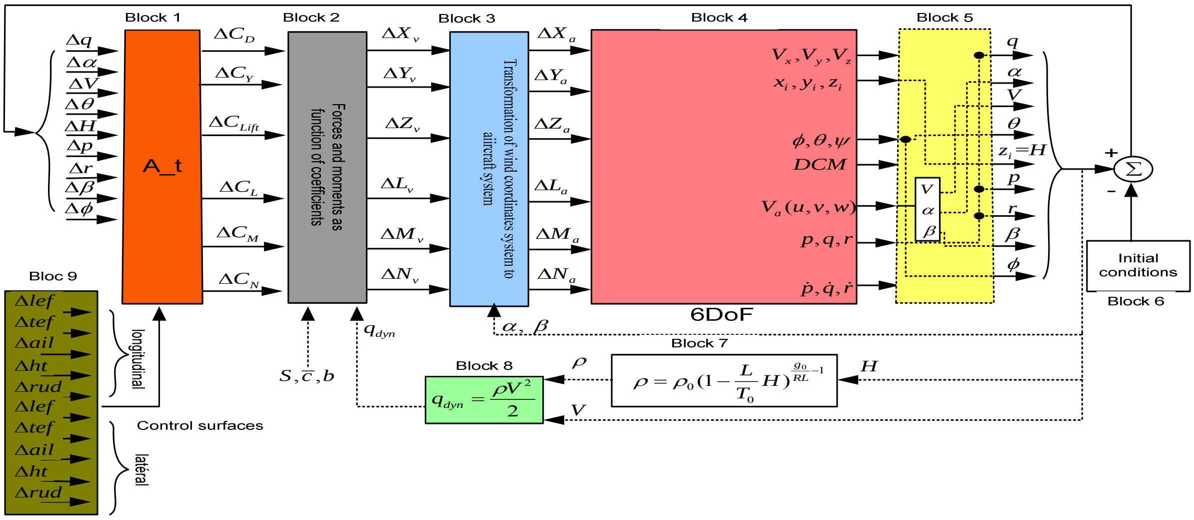

Figure 1: First simulation scheme with the stability and control coefficients calculated in the wind coordinates system

This scheme can be written in its equivalent form shown in Figure 2.

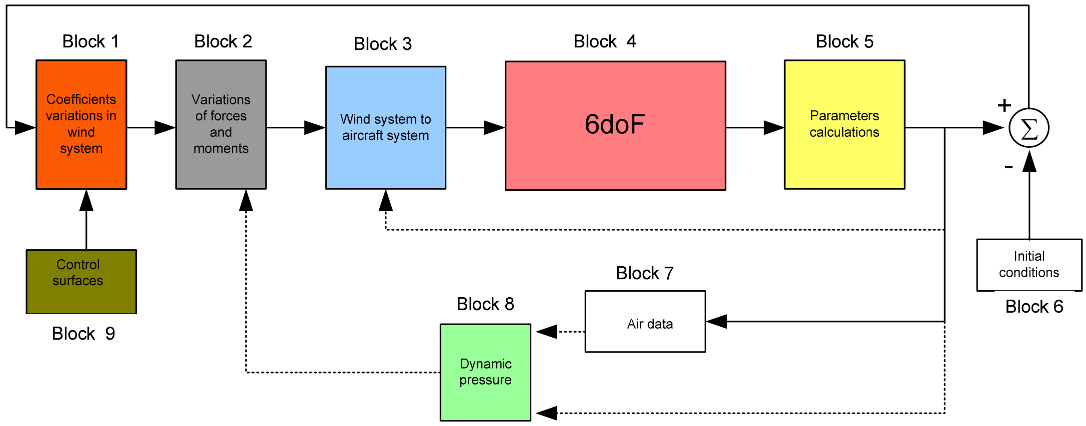

Figure 2: Simplification of the scheme shown in Figure 1

The details of Blocks 1 to 9 are next presented.

Block 1 description

Two sets of stability and control derivatives in the wind coordinate system are known, provided by NASA DFRC (Dryden Flight Research Center) for various flight conditions characterized by Mach numbers, altitudes and angles of attack. We express the aircraft behaviour with the second state space matrix equation:

$$y = \mathrm{A}_{\mathrm{nasa}} x + \mathrm{B}_{\mathrm{nasa}} u = \begin{bmatrix} \mathrm{A}_{\mathrm{nasa}} & \mathrm{B}_{\mathrm{nasa}} \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} = \begin{bmatrix} \mathrm{A\_t} \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} \tag{1}$$

Where the A_t matrix has the dimensions (6*19), and A_{nasa} is the stability derivative coefficients matrix of dimensions (6 x 9):

$$\mathrm{A}_{nasa} = \begin{bmatrix}
\frac{\partial C_L}{\partial q} & \frac{\partial C_L}{\partial \alpha} & \frac{\partial C_L}{\partial V} & \frac{\partial C_L}{\partial \theta} & \frac{\partial C_L}{\partial H} & \frac{\partial C_L}{\partial p} & \frac{\partial C_L}{\partial r} & \frac{\partial C_L}{\partial \beta} & \frac{\partial C_L}{\partial \varphi} \\
\frac{\partial C_M}{\partial q} & \frac{\partial C_M}{\partial \alpha} & \frac{\partial C_M}{\partial V} & \frac{\partial C_M}{\partial \theta} & \frac{\partial C_M}{\partial H} & \frac{\partial C_M}{\partial p} & \frac{\partial C_M}{\partial r} & \frac{\partial C_M}{\partial \beta} & \frac{\partial C_M}{\partial \varphi} \\
\frac{\partial C_N}{\partial q} & \frac{\partial C_N}{\partial \alpha} & \frac{\partial C_N}{\partial V} & \frac{\partial C_N}{\partial \theta} & \frac{\partial C_N}{\partial H} & \frac{\partial C_N}{\partial p} & \frac{\partial C_N}{\partial r} & \frac{\partial C_N}{\partial \beta} & \frac{\partial C_N}{\partial \varphi} \\
\frac{\partial C_D}{\partial q} & \frac{\partial C_D}{\partial \alpha} & \frac{\partial C_D}{\partial V} & \frac{\partial C_D}{\partial \theta} & \frac{\partial C_D}{\partial H} & \frac{\partial C_D}{\partial p} & \frac{\partial C_D}{\partial r} & \frac{\partial C_D}{\partial \beta} & \frac{\partial C_D}{\partial \varphi} \\
\frac{\partial C_{Lift}}{\partial q} & \frac{\partial C_{Lift}}{\partial \alpha} & \frac{\partial C_{Lift}}{\partial V} & \frac{\partial C_{Lift}}{\partial \theta} & \frac{\partial C_{Lift}}{\partial H} & \frac{\partial C_{Lift}}{\partial p} & \frac{\partial C_{Lift}}{\partial r} & \frac{\partial C_{Lift}}{\partial \beta} & \frac{\partial C_{Lift}}{\partial \varphi} \\
\frac{\partial C_Y}{\partial q} & \frac{\partial C_Y}{\partial \alpha} & \frac{\partial C_Y}{\partial V} & \frac{\partial C_Y}{\partial \theta} & \frac{\partial C_Y}{\partial H} & \frac{\partial C_Y}{\partial p} & \frac{\partial C_Y}{\partial r} & \frac{\partial C_Y}{\partial \beta} & \frac{\partial C_Y}{\partial \varphi}
\end{bmatrix} \tag{2}$$

and the control derivative coefficients matrix B_{nasa} has the dimensions (6 x 10) and is divided into two matrices corresponding to the longitudinal and lateral aircraft motion, which are denoted by B_{long_nasa} and B_{lat_nasa}. Therefore, B_{nasa} is written as $\mathrm{B}_{nasa} = \begin{bmatrix} \mathrm{B}_{long\_nasa} & \mathrm{B}_{lat\_nasa} \end{bmatrix}$ where B_{long_nasa} is a (6*6) matrix containing the derivatives of the same coefficients as the ones of A_{nasa} (which are C_L, C_M, C_N, C_D, C_{Lft}, C_Y) with respect to the longitudinal control surfaces δ_{long_AIL}, δ_{long_HT}, δ_{long_RUD}, δ_{long_LEF} and δ_{long_TEF},; B_{lat_nasa} is a (6*6) matrix containing the derivatives of the same coefficients as the ones of A_{nasa} (which are C_L, C_M, C_N, C_D, C_{Lft}, C_Y) with respect to the lateral control surfaces δ_{lat_AIL}, δ_{lat_HT}, δ_{lat_RUD}, δ_{lat_LEF} and δ_{lat}. The output, state, and input vectors are given by the following equations:

$$y = \begin{pmatrix} \Delta C_L & \Delta C_M & \Delta C_N & \Delta C_D & \Delta C_{Lift} & \Delta C_Y \end{pmatrix}^T \tag{3}$$

$$x = \begin{pmatrix} \Delta q & \Delta\alpha & \Delta V & \Delta\theta & \Delta H & \Delta p & \Delta r & \Delta\beta & \Delta\phi \end{pmatrix}^T \tag{4}$$

$$u = \big[\Delta\delta_{long\_LEF} \quad \Delta\delta_{long\_TEF} \quad \Delta\delta_{long\_AIL} \quad \Delta\delta_{long\_HT} \quad \Delta\delta_{long\_RUD} \quad \ldots. \quad \Delta_{lat\_LEF} \quad \Delta_{lat\_TEF} \quad \Delta_{lat\_AIL} \quad \Delta_{lat\_HT} \quad \Delta_{lat\_RUD} \big]^T \tag{5}$$

The Taylor series approximations of stability and control derivatives at the trim position ΔC_L, ΔC_M, ΔC_N, ΔC_D, ΔC_{Lift}, and ΔC_Y can be written as follows:

$$\begin{aligned}\Delta C_L =& \frac{\partial C_L}{\partial q}\Delta q + \frac{\partial C_L}{\partial \alpha}\Delta\alpha + \frac{\partial C_L}{\partial V}\Delta V + \frac{\partial C_L}{\partial \theta}\Delta\theta + \frac{\partial C_L}{\partial H}\Delta H + \frac{\partial C_L}{\partial p}\Delta p + \frac{\partial C_L}{\partial r}\Delta r + \frac{\partial C_L}{\partial \beta}\Delta\beta + \frac{\partial C_L}{\partial \phi}\Delta\phi + \\ & \frac{\partial C_L}{\partial \delta_{Pi\_AIL}}\Delta\delta_{Pi\_AIL} + \frac{\partial C_L}{\partial \delta_{long\_HT}}\Delta\delta_{long\_HT} + \frac{\partial C_L}{\partial \delta_{long\_RUD}}\Delta\delta_{long\_RUD} + \frac{\partial C_L}{\partial \delta_{long\_LEF}}\Delta\delta_{long\_LEF} + \frac{\partial C_L}{\partial \delta_{long\_TEF}}\Delta\delta_{long\_TEF} \\ & + \frac{\partial C_L}{\partial \delta_{lat\_AIL}}\Delta\delta_{lat\_AIL} + \frac{\partial C_L}{\partial \delta_{lat\_HT}}\Delta\delta_{lat\_HT} + \frac{\partial C_L}{\partial \delta_{lat\_RUD}}\Delta\delta_{lat\_RUD} + \frac{\partial C_L}{\partial \delta_{lat\_LEF}}\Delta\delta_{lat\_LEF} + \frac{\partial C_L}{\partial \delta_{lat\_TEF}}\Delta\delta_{lat\_TEF}\end{aligned} \tag{6}$$

The variations of all stability coefficients are written under similar forms as the ones given by equations (6).

Block 2 description

The variations of forces and moments are written as a function of their stability and control derivatives as follows:

$$\begin{pmatrix} \Delta X_w \\ \Delta Y_w \\ \Delta Z_w \\ \Delta L_w \\ \Delta M_w \\ \Delta N_w \end{pmatrix} = \begin{pmatrix} \bar{S} & 0 & 0 & 0 & 0 & 0 \\ 0 & \bar{S} & 0 & 0 & 0 & 0 \\ 0 & 0 & \bar{S} & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{S}\cdot b & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{S}\cdot\bar{c} & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{S}\cdot b \end{pmatrix} \begin{pmatrix} \Delta C_D \\ \Delta C_Y \\ \Delta C_{Lift} \\ \Delta C_L \\ \Delta C_M \\ \Delta C_N \end{pmatrix} \tag{7}$$

where the forces variations on the three axes are $\begin{bmatrix} \Delta X_w & \Delta Y_w & \Delta Z_w \end{bmatrix}$ and the moments variations on all three axes are $\begin{bmatrix} \Delta L_w & \Delta M_w & \Delta N_w \end{bmatrix}$, and where $\bar{S} = \frac{\rho V^2}{2} S = q_{dyn} S$.

Block 3 description

It is well known in this field[4] that an aircraft's system of coordinates (x_a, y_a, z_a) is related to the wind coordinate system (x_w, y_w, z_w) by the following two successive rotations of the coordinate system (x_w, y_w, z_w). 1. A first rotation with sideslip angle β around the z_w axis to obtain the intermediate coordinate system ($x'y'z'$) and 2. A second rotation with attack angle α around the y' axis to obtain the aircraft coordinate system (x_a, y_a, z_a), which gives:

$$\begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix} = \begin{bmatrix} \cos\alpha\cos\beta & -\cos\alpha\sin\beta & -\sin\alpha \\ \sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & -\sin\alpha\sin\beta & \cos\alpha \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} \tag{8}$$

The X_a and Z_a forces have opposite signs (see equations 9.1), while the L_a, M_a, and N_a moments have the same sign (see equations 9.2) with respect to the classical formulation (equations (8)), which is due to their orientations, as given by NASA DFRC.

$$\begin{bmatrix} X_a \\ Y_a \\ Z_a \end{bmatrix} = \begin{bmatrix} -\cos\alpha\cos\beta & \cos\alpha\sin\beta & \sin\alpha \\ \sin\beta & \cos\beta & 0 \\ -\sin\alpha\cos\beta & \sin\alpha\sin\beta & -\cos\alpha \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix} \tag{9.1}$$

$$\begin{bmatrix} L_a \\ M_a \\ N_a \end{bmatrix} = \begin{bmatrix} \cos\alpha\cos\beta & -\cos\alpha\sin\beta & -\sin\alpha \\ \sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & -\sin\alpha\sin\beta & \cos\alpha \end{bmatrix} \begin{bmatrix} L_w \\ M_w \\ N_w \end{bmatrix} \tag{9.2}$$

Equations (9.1) and (9.2) are linearized around the equilibrium position (*trim*) of the F/A-18 aircraft by the small perturbations theory, in which the index of any quantity at equilibrium is denoted by 0 and the variation of a quantity around its equilibrium position is denoted by Δ. The angles of attack and sideslip angles are expressed with this theory as follows:

$$\begin{cases} \alpha = \alpha_0 + \Delta\alpha \\ \beta = \beta_0 + \Delta\beta \end{cases} \tag{10}$$

The sideslip angle at equilibrium β_0 given by the NASA DFRC is equal to 0, and therefore we can write the sideslip angle β as a function of its small variation $\Delta\beta$:

$$\begin{aligned} \cos\beta &= \cos(\beta_0 + \Delta\beta) \simeq \cos\Delta\beta \simeq 1 \\ \sin\beta &= \sin(\beta_0 + \Delta\beta) \simeq \sin\Delta\beta \simeq \Delta\beta \end{aligned} \tag{11}$$

The aircraft forces X, Y, Z and moments L, M, N calculated in the aircraft system a and in the wind system w are also written by use of the small perturbations theory, such as:

$X_a = X_{a0} + \Delta X_a$,…, $L_a = L_{a0} + \Delta L_a$,…, $X_w = X_{w0} + \Delta X_w$,, …, $L_w = L_{w0} + \Delta L_w$ (12)

The forces X_0, Y_0, Z_0 and the moments L_0, M_0, N_0 at equilibrium are zeros in the aircraft system a and in the wind system w, and the angle of attack $\Delta\alpha$ and sideslip angle $\Delta\beta$ variations are also equal to zero. We introduce these last values into equations (9)-(12), and we obtain the following system of equations:

$$\begin{pmatrix} \Delta X_a \\ \Delta Y_a \\ \Delta Z_a \\ \Delta L_a \\ \Delta M_a \\ \Delta N_a \end{pmatrix} = C_{m1} \begin{pmatrix} \Delta X_w \\ \Delta Y_w \\ \Delta Z_w \\ \Delta L_w \\ \Delta M_w \\ \Delta N_w \end{pmatrix} \tag{13}$$

where C_{m1} has the following form :

$$C_{m1} = \begin{pmatrix} -\cos\alpha_0 & 0 & \sin\alpha_0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -\sin\alpha_0 & 0 & -\cos\alpha_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\alpha_0 & 0 & -\sin\alpha_0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \sin\alpha_0 & 0 & \cos\alpha_0 \end{pmatrix} \tag{14}$$

We replace the force and moment variations in the wind system of coordinates ΔX_w, ΔY_w, ΔZ_w, ΔL_w, ΔM_w, and ΔN_w as function of stability coefficients variations given by equation (7) into the right hand side of equation (13) and we obtain:

$$\begin{pmatrix} \Delta X_a \\ \Delta Y_a \\ \Delta Z_a \\ \Delta L_a \\ \Delta M_a \\ \Delta N_a \end{pmatrix} = C_m \begin{pmatrix} \Delta C_D \\ \Delta C_Y \\ \Delta C_{Lift} \\ \Delta C_L \\ \Delta C_M \\ \Delta C_N \end{pmatrix} \tag{15}$$

where the C_m matrix has the following form:

$$C_m = \begin{pmatrix} -\bar{S}\cdot\cos\alpha_0 & 0 & \bar{S}\cdot\sin\alpha_0 & 0 & 0 & 0 \\ 0 & \bar{S} & 0 & 0 & 0 & 0 \\ -\bar{S}\cdot\sin\alpha_0 & 0 & -\bar{S}\cdot\cos\alpha_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{S}\cdot b\cdot\cos\alpha_0 & 0 & -\bar{S}\cdot b\cdot\sin\alpha_0 \\ 0 & 0 & 0 & 0 & \bar{S}\cdot\bar{c} & 0 \\ 0 & 0 & 0 & \bar{S}\cdot b\cdot\sin\alpha_0 & 0 & \bar{S}\cdot b\cdot\cos\alpha_0 \end{pmatrix} \tag{16}$$

Block 4 description

The six degree-of-freedom block refers to the equations of motion of a rigid body in six degrees of freedom. The theory can be found in the literature[4]. The Simulink toolbox is used here as Block 4, and is composed of five blocks, 4.1 to 4.5.

Blocks 4.1 and 4.2 descriptions

The origin of the aircraft system of coordinates, *a*, is the aircraft center of gravity. The inertial system of coordinates is fixed to the Earth and is denoted by *i*. The aircraft equations of motion are obtained with Newton's second law: The sum of the external forces acting on an aircraft is equal to the momentum rate of change of the momentum of the aircraft over time (Block 4.1). The sum of the external moments[4] acting on an aircraft is equal to the angular momentum rate of change of the aircraft over time (Block 4.2).

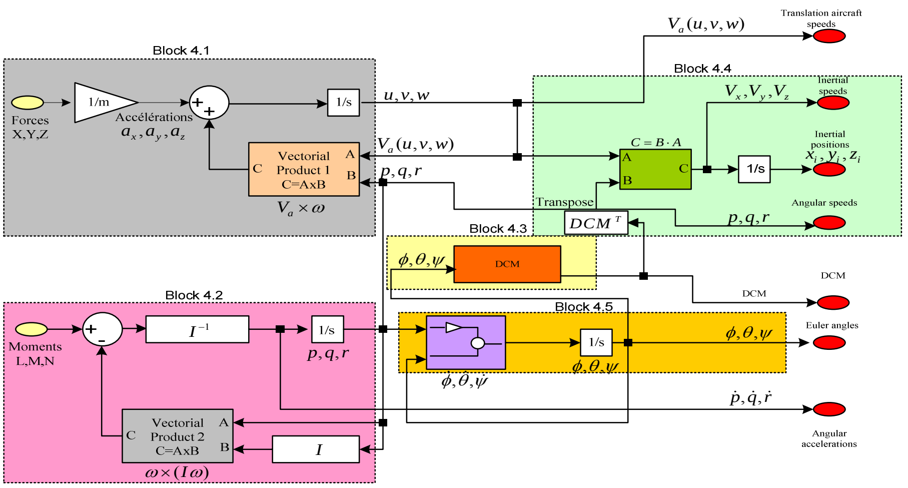

Figure 3: Description of Block 4 details on six degree-of-freedom dynamics

Block 4.3 description

The Cosine Director Matrix CDM is calculated from the Euler roll, pitch and yaw angles ϕ, θ, and ψ in Block 4.3, and is used in Block 4.4.

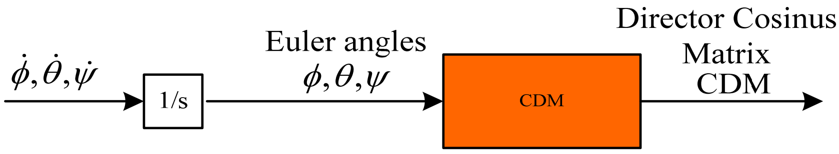

Figure 4: Scheme of Block 4.3

$$\mathrm{CDM}=\begin{pmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \cos\psi\sin\theta\sin\phi-\sin\psi\cos\phi & \sin\phi\sin\theta\sin\psi+\cos\psi\cos\phi & \cos\theta\sin\phi \\ \cos\phi\sin\theta\cos\psi+\sin\phi\sin\psi & \sin\psi\sin\theta\cos\phi-\cos\psi\sin\phi & \cos\theta\cos\phi \end{pmatrix}^T \tag{17}$$

Block 4.4 description

The linear speeds V_x, V_y and V_z in the inertial system of coordinates i are calculated as a function of the linear speeds u, v, and w in the aircraft systems a by three successive rotations: one first rotation with the yaw angle ψ around the z_a axis, a second rotation with the pitch angle θ around the y_a axis and a third rotation with the roll angle ϕ around the x_a axis, and we obtain:

$$\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}=\mathrm{CDM}^{\mathrm{T}}\begin{pmatrix} u \\ v \\ w \end{pmatrix} \tag{18}$$

Block 4.5 description

Block 4.5 relates the Euler angles ϕ, θ, and ψ, and the angular speeds p, q, and r with the Euler time derivatives $\dot{\phi},\dot{\theta}$, and $\dot{\psi}$ by use of the following equation:

$$\begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix}=\begin{pmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \dfrac{\sin\phi}{\cos\theta} & \dfrac{\cos\phi}{\cos\theta} \end{pmatrix}\begin{pmatrix} p \\ q \\ r \end{pmatrix} \tag{19}$$

Block 5 description

In Block 5, we calculate the variations of the aircraft's true airspeed V, of the angle of attack α, and of the sideslip angle β. In order to calculate these variations, we first calculate V, α, and β as functions of linear speeds in translation u, v, and w in the aircraft system of coordinates a[4]:

$$V=\sqrt{u^2+v^2+w^2};\alpha=\arctan(w/u);\beta=\arctan\left[v/\sqrt{(u^2+w^2)}\right] \tag{20}$$

The theory of small perturbations is applied to the terms V, α, β, u, v, and w.

5.1 True airspeed variation ΔV

We replace the speeds given by the perturbation theory in the first squared equation of the system of equations (20):

$$\left(V_0+\Delta V\right)^2=\left(u_0+\Delta u\right)^2+\left(v_0+\Delta v\right)^2+\left(w_0+\Delta w\right)^2 \tag{21}$$

At equilibrium we write:

$$V_0^2=u_0^2+v_0^2+w_0^2 \tag{22}$$

The square products of speed variations Δu^2, Δv^2, Δw^2, and ΔV^2 in equation (21) can be neglected, as they are very small. Then, we divide equation (21) by $2V_0$ and we obtain:

$$\Delta V = \frac{u_0}{V_0}\Delta u + \frac{v_0}{V_0}\Delta v + \frac{w_0}{V_0}\Delta w \tag{23}$$

At equilibrium, the components u_0 and w_0 of the true airspeed V_0 can be written as a function of the angle of attack α_0 as follows:

$$u_0 = V_0 \cos\alpha_0; \quad w_0 = V_0 \sin\alpha_0 \tag{24}$$

The researchers at NASA DFRC considered in their calculations $\beta_0 = 0$, so that the component v_0 is equal to zero:

$$v_0 = V_0 \sin\beta_0 = 0 \tag{25}$$

We replace equations (24) and (25) into equation (23) and we obtain:

$$\Delta V = \Delta u \cos\alpha_0 + \Delta w \sin\alpha_0 \tag{26}$$

5.2 Angle of attack variation $\Delta\alpha$

The second equation of system (20) can also be written in the following form:

$$u \sin\alpha = w \cos\alpha \tag{27}$$

At equilibrium, equation (27) can be expressed as:

$$u_0 \sin\alpha_0 = w_0 \cos\alpha_0 \tag{28}$$

We apply the perturbation theory to the quantities u, v and α (for example $u = u_0 + \Delta u$, etc.). We further replace these quantities, equations (24) and (28) in equation (27). The products of small quantity variations, such as $\Delta\alpha\,\Delta u$ and $\Delta\alpha\,\Delta w$ are neglected. We take into account the trigonometric equality $\sin^2\alpha_0 + \cos^2\alpha_0 = 1$ and the trigonometric functions for small angles of attack variations ($\cos\Delta\alpha = 1$ and $\sin\Delta\alpha = \Delta\alpha$), therefore we obtain the variation of the angle of attack as follows:

$$\Delta\alpha = \frac{-\sin\alpha_0}{V_0}\Delta u + \frac{\cos\alpha_0}{V_0}\Delta w \tag{29}$$

5.3 Sideslip angle variation $\Delta\beta$

The third equation of the system of equations (20) can be written, for small variations of $\Delta\beta_j$ (and for $\beta_0 = 0$), as :

$$\tan\Delta\beta \simeq \Delta\beta = \frac{v}{\sqrt{u^2 + w^2}} \tag{30}$$

We express $\Delta\beta$ in the form of the products sum of the β derivatives with respect to u, v, and w at equilibrium and the small perturbations of u, v, and w:

$$\Delta\beta = \left(\frac{\partial\beta}{\partial u}\right)_0 \Delta u + \left(\frac{\partial\beta}{\partial v}\right)_0 \Delta v + \left(\frac{\partial\beta}{\partial w}\right)_0 \Delta w \tag{31}$$

The expressions of β derivatives at equilibrium are calculated by the derivation of equation (45), where $v_0 = 0$, $V_0^2 = u_0^2 + w_0^2$, and we obtain:

$$\begin{aligned}
\left(\frac{\partial\beta}{\partial u}\right)_0 &= \frac{1}{1 + \dfrac{v_0^2}{u_0^2 + w_0^2}} \cdot v_0 \cdot \left[-\frac{1}{2}\right] \frac{1}{(\sqrt{u_0^2 + w_0^2})^3} \cdot 2u_0 = 0 \\
\left(\frac{\partial\beta}{\partial v}\right)_0 &= \frac{1}{1 + \dfrac{v_0^2}{u_0^2 + w_0^2}} \cdot \frac{1}{\sqrt{u_0^2 + w_0^2}} = \frac{\sqrt{u_0^2 + w_0^2}}{u_0^2 + w_0^2 + v_0^2} = \frac{\sqrt{u_0^2 + w_0^2}}{u_0^2 + w_0^2} = \frac{1}{V_0} \\
\left(\frac{\partial\beta}{\partial w}\right)_0 &= \frac{1}{1 + \dfrac{v_0^2}{u_0^2 + w_0^2}} \cdot v_0 \cdot \left[-\frac{1}{2}\right] \frac{1}{(\sqrt{u_0^2 + w_0^2})^3} \cdot 2w_0 = 0
\end{aligned} \tag{32}$$

We replace the system of equations (32) in equation (31) and we obtain:

$$\Delta\beta = \frac{1}{V_0}\Delta v \tag{33}$$

Finally, ΔV, $\Delta\alpha$, and $\Delta\beta$, from equations (26), (29) and (33), respectively, are rearranged in the form of a matrix system of equations:

$$\begin{pmatrix} \Delta V \\ \Delta\alpha \\ \Delta\beta \end{pmatrix} = \begin{pmatrix} \cos\alpha_0 & 0 & \sin\alpha_0 \\ \frac{-\sin\alpha_0}{V_0} & 0 & \frac{\cos\alpha_0}{V_0} \\ 0 & \frac{1}{V_0} & 0 \end{pmatrix} \begin{pmatrix} \Delta u \\ \Delta v \\ \Delta w \end{pmatrix} \tag{34}$$

We add the system of equations (34) to the initial equations of the Block 4 output variations Δu, Δv, Δw, Δx_i, Δy_i, Δz_i, Δp, Δq, Δr, $\Delta\phi$, $\Delta\theta$, and $\Delta\psi$, which become the Block 5 inputs, and we obtain a higher order matrix system of equations:

$$\begin{pmatrix} \Delta q \\ \Delta\alpha \\ \Delta V \\ \Delta\theta \\ \Delta H \\ \Delta p \\ \Delta r \\ \Delta\beta \\ \Delta\phi \end{pmatrix} = B_m \begin{pmatrix} \Delta u \\ \Delta v \\ \Delta w \\ \Delta x_i \\ \Delta y_i \\ \Delta z_i = \Delta H \\ \Delta p \\ \Delta q \\ \Delta r \\ \Delta\phi \\ \Delta\theta \\ \Delta\psi \end{pmatrix} \tag{35}$$

where

$$B_m = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -\frac{\sin\alpha_0}{V_0} & 0 & \frac{\cos\alpha_0}{V_0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cos\alpha_0 & 0 & \sin\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1}{V_0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \tag{36}$$

Block 6 description

The initial parameters of the matrix system of equations (35) are given by NASA DFRC in the stability and control derivative files for each flight condition characterized by the Mach number, altitude and the angle of attack:

$$\begin{bmatrix} q_0 & \alpha_0 & V_0 & \theta_0 & H_0 & p_0 & r_0 & \beta_0 & \phi_0 \end{bmatrix}^T = \begin{bmatrix} 0 & \alpha_{0\_nasa} & V_{0\_nasa} & \theta_{0\_nasa} & H_{0\_nasa} & 0 & 0 & 0 & 0 \end{bmatrix}^T \tag{37}$$

Block 7 description
We calculate the air density ρ as a function of altitude H, and we obtain[5]:

$$\frac{\rho}{\rho_0} = \left(1 - \frac{LH}{T_0}\right)^{\frac{g_0}{LR}-1} \tag{38}$$

where ρ_0 is the air density at sea level, L is the temperature lapse rate, T_0 is the air temperature at sea level, H is the altitude, R is the gas constant and g_0 is the gravitational constant at sea level.
Block 8 description
The dynamic pressure q_{dyn} is calculated[4] as a function of the aircraft's true airspeed V and the air density ρ.
Block 9 description
The control surface inputs are excited with various signals. The control surfaces for the F/A-18 aircraft analyzed here are: the Leading Edge LE flaps $\Delta lef = \Delta\delta_{LEF}$, the Trailing Edge TE flaps $\Delta tef = \Delta\delta_{TEF}$, the ailerons $\Delta ail = \Delta\delta_{AIL}$, the horizontal tail $\Delta ht = \Delta\delta_{HT}$, and the rudder $\Delta rud = \Delta\delta_{RUD}$. One signal is used at a time on the aircraft control inputs, to give a deviation of $\pm 5^0$ around the equilibrium aircraft position (*trim*) for its longitudinal and lateral aircraft motion in order to observe the forces and moments variation over time. For the longitudinal motion, a signal was given on the horizontal tail, and for the lateral motion a signal was given on the ailerons.

2.2 STATE VARIABLES INTRODUCTION IN CLOSED LOOP

We next develop a second formulation from the first formulation, in order to obtain the vectors of the generalized coordinates and their time derivatives in closed loop form:

$$\eta = (\Delta x_i, \Delta y_i, \Delta z_i, \Delta\phi, \Delta\theta, \Delta\psi)^T; \quad \dot{\eta} = (\Delta V_x, \Delta V_y, \Delta V_z, \Delta\dot{\phi}, \Delta\dot{\theta}, \Delta\dot{\psi})^T \tag{39}$$

The arrangement of stability coefficients on lines 1 to 6 of the matrices used in the first formulation is: $C_L, C_M, C_N, C_D, C_{Lift}, C_Y$ (see equation (7)). In order to obtain the second formulation, it is necessary to rearrange the order of the stability coefficients as follows: $C_D, C_Y, C_{Lift}, C_L, C_M, C_N$. Therefore, the notations A_{int}, B_{long_int}, and B_{lat_int} are introduced for the intermediate matrices A, B_{long}, and B_{lat}, in which the order of the coefficients is rearranged.

We show below only the A_{int} matrix, while B_{long_int} (6*6) matrix contains the derivatives of the same coefficients as the ones of A_{int} with respect to the longitudinal control surfaces δ_{long_AIL}, δ_{long_HT}, δ_{long_RUD}, δ_{long_LEF} and δ_{long_TEF}, and B_{lat_nasa} is a (6*6) matrix containing the derivatives of the same coefficients as the ones of A_{int} with respect to the lateral control surfaces δ_{lat_AIL}, δ_{lat_HT}, δ_{lat_RUD}, δ_{lat_LEF} and δ_{lat_TEF}.

$$A_{int} = \begin{bmatrix} \frac{\partial C_D}{\partial q} & \frac{\partial C_D}{\partial \alpha} & \frac{\partial C_D}{\partial V} & \frac{\partial C_D}{\partial \theta} & \frac{\partial C_D}{\partial H} & \frac{\partial C_D}{\partial p} & \frac{\partial C_D}{\partial r} & \frac{\partial C_D}{\partial \beta} & \frac{\partial C_D}{\partial \varphi} \\ \frac{\partial C_Y}{\partial q} & \frac{\partial C_Y}{\partial \alpha} & \frac{\partial C_Y}{\partial V} & \frac{\partial C_Y}{\partial \theta} & \frac{\partial C_Y}{\partial H} & \frac{\partial C_Y}{\partial p} & \frac{\partial C_Y}{\partial r} & \frac{\partial C_Y}{\partial \beta} & \frac{\partial C_Y}{\partial \varphi} \\ \frac{\partial C_{Lift}}{\partial q} & \frac{\partial C_{Lift}}{\partial \alpha} & \frac{\partial C_{Lift}}{\partial V} & \frac{\partial C_{Lift}}{\partial \theta} & \frac{\partial C_{Lift}}{\partial H} & \frac{\partial C_{Lift}}{\partial p} & \frac{\partial C_{Lift}}{\partial r} & \frac{\partial C_{Lift}}{\partial \beta} & \frac{\partial C_{Lift}}{\partial \varphi} \\ \frac{\partial C_L}{\partial q} & \frac{\partial C_L}{\partial \alpha} & \frac{\partial C_L}{\partial V} & \frac{\partial C_L}{\partial \theta} & \frac{\partial C_L}{\partial H} & \frac{\partial C_L}{\partial p} & \frac{\partial C_L}{\partial r} & \frac{\partial C_L}{\partial \beta} & \frac{\partial C_L}{\partial \varphi} \\ \frac{\partial C_M}{\partial q} & \frac{\partial C_M}{\partial \alpha} & \frac{\partial C_M}{\partial V} & \frac{\partial C_M}{\partial \theta} & \frac{\partial C_M}{\partial H} & \frac{\partial C_M}{\partial p} & \frac{\partial C_M}{\partial r} & \frac{\partial C_M}{\partial \beta} & \frac{\partial C_M}{\partial \varphi} \\ \frac{\partial C_N}{\partial q} & \frac{\partial C_N}{\partial \alpha} & \frac{\partial C_N}{\partial V} & \frac{\partial C_N}{\partial \theta} & \frac{\partial C_N}{\partial H} & \frac{\partial C_N}{\partial p} & \frac{\partial C_N}{\partial r} & \frac{\partial C_N}{\partial \beta} & \frac{\partial C_N}{\partial \varphi} \end{bmatrix} \tag{40}$$

The intermediate matrix A_{m_int} for the A_m matrix is now written as follows:

$$A_{m\_\text{int}} = \begin{bmatrix} A_{\text{int}} & B_{long\_\text{int}} & B_{lat\_\text{int}} \end{bmatrix} = \begin{bmatrix} a_{4,1} & a_{4,2} & \cdots & a_{4,19} \\ a_{6,1} & a_{6,2} & \cdots & a_{6,19} \\ a_{5,1} & a_{5,2} & \cdots & a_{5,19} \\ a_{1,1} & a_{1,2} & \cdots & a_{1,19} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,19} \\ a_{3,1} & a_{3,2} & \cdots & a_{3,19} \end{bmatrix} = \begin{bmatrix} A_{m1\_\text{int}} & A_{m2\_\text{int}} \end{bmatrix} \tag{41}$$

where A_{m1_int} is the stability coefficients matrix and its analytical formulation is given in equation (42.1). The A_{m2_int} matrix is the control coefficients matrix $[B_{long\_\text{int}} \;\; B_{lat\_\text{int}}]$ and its analytical form is expressed by equation (42.2).

$$A_{m1\_\text{int}} = \begin{bmatrix} a_{4,1} & a_{4,2} & \cdots & a_{4,9} \\ a_{6,1} & a_{6,2} & \cdots & a_{6,9} \\ a_{5,1} & a_{5,2} & \cdots & a_{5,9} \\ a_{1,1} & a_{1,2} & \cdots & a_{1,9} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,9} \\ a_{3,1} & a_{3,2} & \cdots & a_{3,9} \end{bmatrix} \tag{42.1}$$

$$A_{m2\_\text{int}} = \begin{bmatrix} B_{long\_\text{int}} & B_{lat\_\text{int}} \end{bmatrix} = \begin{bmatrix} a_{4,10} & a_{4,11} & \cdots & a_{4,19} \\ a_{6,10} & a_{6,11} & \cdots & a_{6,19} \\ a_{5,10} & a_{5,11} & \cdots & a_{5,19} \\ a_{1,10} & a_{1,11} & \cdots & a_{1,19} \\ a_{2,10} & a_{2,11} & \cdots & a_{2,19} \\ a_{3,10} & a_{3,11} & \cdots & a_{3,19} \end{bmatrix} \tag{42.2}$$

Then, the second equation of a state space system may be written by use of equations (42.1) and (42.2) in the rearranged order, as follows:

$$\begin{pmatrix} \Delta C_D \\ \Delta C_Y \\ \Delta C_{Lift} \\ \Delta C_L \\ \Delta C_M \\ \Delta C_N \end{pmatrix} = A_{m1\_\text{int}} \begin{pmatrix} \Delta q \\ \Delta \alpha \\ \Delta V \\ \Delta \theta \\ \Delta H \\ \Delta p \\ \Delta r \\ \Delta \beta \\ \Delta \phi \end{pmatrix} + A_{m2\_\text{int}} \begin{pmatrix} \Delta \delta_{long\_LEF} \\ \Delta \delta_{long\_TEF} \\ \Delta \delta_{long\_AIL} \\ \Delta \delta_{long\_HT} \\ \Delta \delta_{long\_RUD} \\ \Delta \delta_{lat\_LEF} \\ \Delta \delta_{lat\_TEF} \\ \Delta \delta_{lat\_AIL} \\ \Delta \delta_{lat\_HT} \\ \Delta \delta_{lat\_RUD} \end{pmatrix} \tag{43}$$

We replace the left hand side term of equation (35) in the first term on the right hand side of equation (43) in order to obtain another set of state vectors **x** in the second equation of the state space system of equations, and we obtain:

$$\begin{pmatrix} \Delta C_D \\ \Delta C_Y \\ \Delta C_{Lift} \\ \Delta C_L \\ \Delta C_M \\ \Delta C_N \end{pmatrix} = A_{m1\_int} B_m \begin{pmatrix} \Delta u \\ \Delta v \\ \Delta w \\ \Delta x_i \\ \Delta y_i \\ \Delta z_i = \Delta H \\ \Delta p \\ \Delta q \\ \Delta r \\ \Delta \phi \\ \Delta \theta \\ \Delta \psi \end{pmatrix} + A_{m2\_int} \begin{pmatrix} \Delta \delta_{long\_LEF} \\ \Delta \delta_{long\_TEF} \\ \Delta \delta_{long\_AIL} \\ \Delta \delta_{long\_HT} \\ \Delta \delta_{long\_RUD} \\ \Delta \delta_{lat\_LEF} \\ \Delta \delta_{lat\_TEF} \\ \Delta \delta_{lat\_AIL} \\ \Delta \delta_{lat\_HT} \\ \Delta \delta_{lat\_RUD} \end{pmatrix} \tag{44}$$

We replace the vector output given on the left hand side of equation (44) in the right hand side term of equation (15) in order to calculate the forces and moments in the aircraft system of coordinates:

$$\begin{pmatrix} \Delta X_a \\ \Delta Y_a \\ \Delta Z_a \\ \Delta L_a \\ \Delta M_a \\ \Delta N_a \end{pmatrix} = C_m A_{m1\_int} B_m \begin{pmatrix} \Delta u \\ \Delta v \\ \Delta w \\ \Delta x_i \\ \Delta y_i \\ \Delta z_i = \Delta H \\ \Delta p \\ \Delta q \\ \Delta r \\ \Delta \phi \\ \Delta \theta \\ \Delta \psi \end{pmatrix} + C_m A_{m2\_int} \begin{pmatrix} \Delta \delta_{long\_LEF} \\ \Delta \delta_{long\_TEF} \\ \Delta \delta_{long\_AIL} \\ \Delta \delta_{long\_HT} \\ \Delta \delta_{long\_RUD} \\ \Delta \delta_{lat\_LEF} \\ \Delta \delta_{lat\_TEF} \\ \Delta \delta_{lat\_AIL} \\ \Delta \delta_{lat\_HT} \\ \Delta \delta_{lat\_RUD} \end{pmatrix} \tag{45}$$

The following notations are introduced:

$$C = C_m A_{m1\_int} B_m; \qquad D = C_m A_{m2\_int} \tag{46}$$

and we obtain:

$$\begin{pmatrix} \Delta X_a \\ \Delta Y_a \\ \Delta Z_a \\ \Delta L_a \\ \Delta M_a \\ \Delta N_a \end{pmatrix} = C \begin{pmatrix} \Delta u \\ \Delta v \\ \Delta w \\ \Delta x_i \\ \Delta y_i \\ \Delta z_i = H \\ \Delta p \\ \Delta q \\ \Delta r \\ \Delta \phi \\ \Delta \theta \\ \Delta \psi \end{pmatrix} + D \begin{pmatrix} \Delta \delta_{long\_LEF} \\ \Delta \delta_{long\_TEF} \\ \Delta \delta_{long\_AIL} \\ \Delta \delta_{long\_HT} \\ \Delta \delta_{long\_RUD} \\ \Delta \delta_{lat\_LEF} \\ \Delta \delta_{lat\_TEF} \\ \Delta \delta_{lat\_AIL} \\ \Delta \delta_{lat\_HT} \\ \Delta \delta_{lat\_RUD} \end{pmatrix} \tag{47}$$

The scheme shown in the next figure represents the conversion of the scheme shown in Figure 1 by use of equations (39)-(47), and has the 12 state vectors x given by equation (39) in the closed loop. We obtained the *C* and *D* matrices, which characterize the linear system at trim condition. These matrices are presented in detail next.

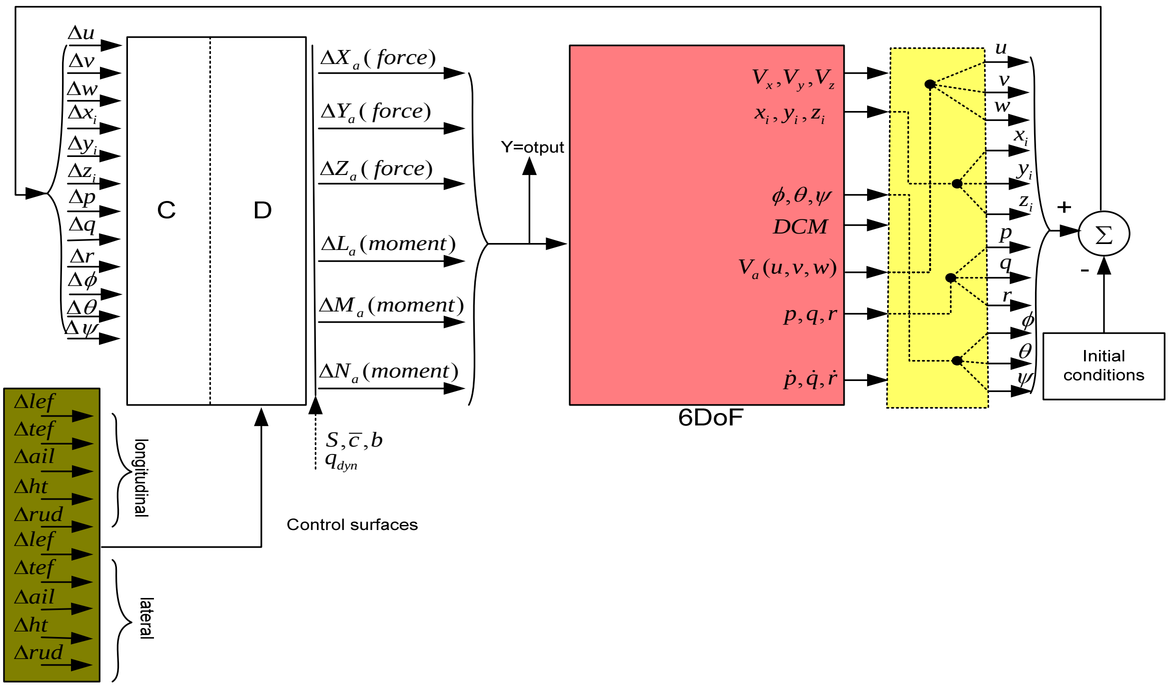

Figure 5: Simulation scheme with 12 state vectors in closed loop

We can represent the terms of the C and D matrices analytically. The C matrix has the following analytical form:

$$C = C_m A_{m1} B_m = \begin{pmatrix} C11 & C12 & C13 & C14 \\ C21 & C22 & C23 & C24 \end{pmatrix} \tag{48}$$

where the $C11$, $C12$, $C13$, $C14$, $C21$, $C22$, $C23$, and $C24$ matrices are represented under analytical forms. The B_m matrix is expressed by equation (51), the A_{m1_int} matrix is given by equation (42.1), and their product is written as follows:

$$A_{m1}B_m = \begin{pmatrix}
-a_{4,2}\frac{\sin\alpha_0}{V_0} + a_{4,3}\cos\alpha_0 & \frac{a_{4,8}}{V_0} & a_{4,2}\frac{\cos\alpha_0}{V_0} + a_{4,3}\sin\alpha_0 & 0 & 0 & a_{4,5} & a_{4,6} & a_{4,1} & a_{4,7} & a_{4,9} & a_{4,4} & 0 \\
-a_{6,2}\frac{\sin\alpha_0}{V_0} + a_{6,3}\cos\alpha_0 & \frac{a_{6,8}}{V_0} & a_{6,2}\frac{\cos\alpha_0}{V_0} + a_{6,3}\sin\alpha_0 & 0 & 0 & a_{6,5} & a_{6,6} & a_{6,1} & a_{6,7} & a_{6,9} & a_{6,4} & 0 \\
-a_{5,2}\frac{\sin\alpha_0}{V_0} + a_{5,3}\cos\alpha_0 & \frac{a_{5,8}}{V_0} & a_{5,2}\frac{\cos\alpha_0}{V_0} + a_{5,3}\sin\alpha_0 & 0 & 0 & a_{5,5} & a_{5,6} & a_{5,1} & a_{5,7} & a_{5,9} & a_{5,4} & 0 \\
-a_{1,2}\frac{\sin\alpha_0}{V_0} + a_{1,3}\cos\alpha_0 & \frac{a_{1,8}}{V_0} & a_{1,2}\frac{\cos\alpha_0}{V_0} + a_{1,3}\sin\alpha_0 & 0 & 0 & a_{1,5} & a_{1,6} & a_{1,1} & a_{1,7} & a_{1,9} & a_{1,4} & 0 \\
-a_{2,2}\frac{\sin\alpha_0}{V_0} + a_{2,3}\cos\alpha_0 & \frac{a_{2,8}}{V_0} & a_{2,2}\frac{\cos\alpha_0}{V_0} + a_{2,3}\sin\alpha_0 & 0 & 0 & a_{2,5} & a_{2,6} & a_{2,1} & a_{2,7} & a_{2,9} & a_{2,4} & 0 \\
-a_{3,2}\frac{\sin\alpha_0}{V_0} + a_{3,3}\cos\alpha_0 & \frac{a_{3,8}}{V_0} & a_{3,2}\frac{\cos\alpha_0}{V_0} + a_{3,3}\sin\alpha_0 & 0 & 0 & a_{3,5} & a_{3,6} & a_{3,1} & a_{3,7} & a_{3,9} & a_{3,4} & 0
\end{pmatrix} \tag{49}$$

The C matrix is the product of the C_m matrix given by equation (24) and the $A_{m1}B_m$ matrices' product given by equation (49), while the C matrix analytical elements are found by identification. Due to the pages number limitations, we give here only the C11 matrix elements:

$$c_{1,1} = \bar{S}\left(\frac{a_{4,2}\sin 2\alpha_0}{2V_0} - a_{4,3}\cos^2\alpha_0 - \frac{a_{5,2}\sin^2\alpha_0}{V_0} + \frac{a_{5,3}\sin 2\alpha_0}{2}\right); \quad c_{1,2} = \bar{S}\left(-\frac{a_{4,8}\cos\alpha_0}{V_0} + \frac{a_{5,8}\sin\alpha_0}{V_0}\right)$$

$$c_{1,3} = \bar{S}\left(\frac{-a_{4,2}\cos^2\alpha_0}{V_0} - \frac{a_{4,3}\sin 2\alpha_0}{2} + \frac{a_{5,2}\sin 2\alpha_0}{2V_0} + a_{5,3}\sin^2\alpha_0\right); \tag{50}$$

$$c_{2,1} = \bar{S}\cdot\left(-\frac{a_{6,2}\cdot\sin\alpha_0}{V_0} + a_{6,3}\cdot\cos\alpha_0\right);\ c_{2,2} = \bar{S}\cdot\frac{a_{6,8}}{V_0};\ c_{2,3} = \bar{S}\cdot\left(\frac{a_{6,2}\cdot\cos\alpha_0}{V_0} + a_{6,3}\cdot\sin\alpha_0\right)$$

$$c_{3,1} = \bar{S}\left(\frac{a_{4,2}\sin^2\alpha_0}{V_0} - \frac{a_{4,3}\sin 2\alpha_0}{2} + \frac{a_{5,2}\sin 2\alpha_0}{2V_0} - a_{5,3}\cos^2\alpha_0\right);\quad c_{3,2} = \bar{S}\left(-\frac{a_{4,8}\sin\alpha_0}{V_0} - \frac{a_{5,8}\cos\alpha_0}{V_0}\right)$$

The D matrix may be written under the form $D = \begin{bmatrix} D1 & D2 \end{bmatrix}^T$, where $D1$ and $D2$ are:

$$D1 = \begin{pmatrix} d_{1,1} & d_{1,2} & d_{1,3} & d_{1,4} & d_{1,5} & d_{1,6} & d_{1,7} & d_{1,8} & d_{1,9} & d_{1,10} \\ d_{2,1} & d_{2,2} & d_{2,3} & d_{2,4} & d_{2,5} & d_{2,6} & d_{2,7} & d_{2,8} & d_{2,9} & d_{2,10} \\ d_{3,1} & d_{3,2} & d_{3,3} & d_{3,4} & d_{3,5} & d_{3,6} & d_{3,7} & d_{3,8} & d_{3,9} & d_{3,10} \end{pmatrix} \tag{51.1}$$

$$D2 = \begin{pmatrix} d_{4,1} & d_{4,2} & d_{4,3} & d_{4,4} & d_{4,5} & d_{4,6} & d_{4,7} & d_{4,8} & d_{4,9} & d_{4,10} \\ d_{5,1} & d_{5,2} & d_{5,3} & d_{5,4} & d_{5,5} & d_{5,6} & d_{5,7} & d_{5,8} & d_{5,9} & d_{5,10} \\ d_{6,1} & d_{6,2} & d_{6,3} & d_{6,4} & d_{6,5} & d_{6,6} & d_{6,7} & d_{6,8} & d_{6,9} & d_{6,10} \end{pmatrix} \tag{51.2}$$

The matrix $D = \begin{bmatrix} D1 & D2 \end{bmatrix}^T = C_m A_{m2\_\mathrm{int}}$ is the product of the C_m matrix given by equation (16) and the A_{m2_int} given by equation (42.2). The elements of matrices $D1$ and $D2$ are calculated by identification, and we show here only the first elements of $D1$ matrix.

$$\begin{aligned} d_{1,1} &= \bar{S}\cdot\left(-a_{4,10}\cdot\cos\alpha_0 + a_{5,10}\cdot\sin\alpha_0\right); d_{1,2} = \bar{S}\cdot\left(-a_{4,11}\cdot\cos\alpha_0 + a_{5,11}\cdot\sin\alpha_0\right) \\ d_{1,3} &= \bar{S}\cdot\left(-a_{4,12}\cdot\cos\alpha_0 + a_{5,12}\cdot\sin\alpha_0\right); d_{1,4} = \bar{S}\cdot\left(-a_{4,13}\cdot\cos\alpha_0 + a_{5,13}\cdot\sin\alpha_0\right) \\ d_{1,5} &= \bar{S}\cdot\left(-a_{4,14}\cdot\cos\alpha_0 + a_{5,14}\cdot\sin\alpha_0\right); d_{1,6} = \bar{S}\cdot\left(-a_{4,15}\cdot\cos\alpha_0 + a_{5,15}\cdot\sin\alpha_0\right) \\ d_{1,7} &= \bar{S}\cdot\left(-a_{4,16}\cdot\cos\alpha_0 + a_{5,16}\cdot\sin\alpha_0\right); d_{1,8} = \bar{S}\cdot\left(-a_{4,17}\cdot\cos\alpha_0 + a_{5,17}\cdot\sin\alpha_0\right) \\ d_{1,9} &= \bar{S}\cdot\left(-a_{4,18}\cdot\cos\alpha_0 + a_{5,18}\cdot\sin\alpha_0\right); d_{1,10} = \bar{S}\cdot\left(-a_{4,19}\cdot\cos\alpha_0 + a_{5,19}\cdot\sin\alpha_0\right) \end{aligned} \tag{52}$$

The C matrix (only its C11 elements are shown in equations (50)) and the D matrix (only its first elements are shown in equations (52)) are further replaced in equation (47). Therefore, we obtain the two matrix equations for the variations of forces ΔX_a, ΔY_a et ΔZ_a and moments ΔL_a, ΔM_a et ΔN_a in the aircraft system of coordinates:

$$\begin{pmatrix} \Delta X_a \\ \Delta Y_a \\ \Delta Z_a \end{pmatrix} = C11\begin{pmatrix} \Delta u \\ \Delta v \\ \Delta w \end{pmatrix} + C12\begin{pmatrix} \Delta x_i \\ \Delta y_i \\ \Delta z_i \end{pmatrix} + C13\begin{pmatrix} \Delta p \\ \Delta q \\ \Delta r \end{pmatrix} + C14\begin{pmatrix} \Delta\phi \\ \Delta\theta \\ \Delta\psi \end{pmatrix} + D1\begin{pmatrix} \Delta\delta_{long\_LEF} \\ \Delta\delta_{long\_TEF} \\ \Delta\delta_{long\_AIL} \\ \Delta\delta_{long\_HT} \\ \Delta\delta_{long\_RUD} \\ \Delta\delta_{lat\_LEF} \\ \Delta\delta_{lat\_TEF} \\ \Delta\delta_{lat\_AIL} \\ \Delta\delta_{lat\_HT} \\ \Delta\delta_{lat\_RUD} \end{pmatrix} \tag{53.1}$$

$$\begin{pmatrix} \Delta L_a \\ \Delta M_a \\ \Delta N_a \end{pmatrix} = C21 \begin{pmatrix} \Delta u \\ \Delta v \\ \Delta w \end{pmatrix} + C22 \begin{pmatrix} \Delta x_i \\ \Delta y_i \\ \Delta z_i \end{pmatrix} + C23 \begin{pmatrix} \Delta p \\ \Delta q \\ \Delta r \end{pmatrix} + C24 \begin{pmatrix} \Delta \phi \\ \Delta \theta \\ \Delta \psi \end{pmatrix} + D2 \begin{pmatrix} \Delta\delta_{long\_LEF} \\ \Delta\delta_{long\_TEF} \\ \Delta\delta_{long\_AIL} \\ \Delta\delta_{long\_HT} \\ \Delta\delta_{long\_RUD} \\ \Delta\delta_{lat\_LEF} \\ \Delta\delta_{lat\_TEF} \\ \Delta\delta_{lat\_AIL} \\ \Delta\delta_{lat\_HT} \\ \Delta\delta_{lat\_RUD} \end{pmatrix} \tag{53.2}$$

In equations (53.1) and (53.2), there are terms u, v, w, p, q, and r in the aircraft coordinate system and terms x_i, y_i, z_i, ϕ, θ, and ψ in the inertial coordinate system. We need to obtain these terms in the same system of coordinates, and we chose the inertial system of coordinates. Therefore, we convert the linear and angular speeds terms (u, v, w, p, q, and r) from the aircraft coordinates system a, into the inertial coordinate system i. The speeds u, v, and w in the aircraft system of coordinates a are obtained, using linearization, from the linear speeds and Euler angles in the inertial system of coordinates i. Equation (18) can be written in the following form:

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix}_a = \begin{pmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \sin\phi\cos\theta \\ \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi & \cos\phi\cos\theta \end{pmatrix} \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}_i \tag{54}$$

The small perturbations theory is applied to the following quantities: u, v, w, V_x, V_y, V_z, p, q, and r. The roll and yaw angles at equilibrium given by NASA DFRC are equal to zero, ϕ_0, and ψ_0. The trigonometric function (**sine** and **cosine**) assumptions for small angles $\Delta\phi$, $\Delta\theta$, and $\Delta\psi$ are also considered. The products of small terms such as $\Delta\phi\Delta\psi$, $\Delta\theta\Delta V_x$, $\Delta\theta\Delta V_z$, $\Delta\phi\Delta V_x$ and $\Delta\phi\Delta V_y$ are neglected, and we consider the equilibrium speeds V_{y_0} and V_{z_0} to be zero. We replace these assumptions in equations (54) and we obtain:

$$\begin{pmatrix} \Delta u \\ \Delta v \\ \Delta w \end{pmatrix} = P \begin{pmatrix} \Delta V_x \\ \Delta V_y \\ \Delta V_z \end{pmatrix} + P1 \begin{pmatrix} \Delta\phi \\ \Delta\theta \\ \Delta\psi \end{pmatrix} \tag{55}$$

where:

$$P = \begin{pmatrix} \cos\theta_0 & 0 & -\sin\theta_0 \\ 0 & 1 & 0 \\ \sin\theta_0 & 0 & \cos\theta_0 \end{pmatrix} \text{ and } P1 = \begin{pmatrix} 0 & -V_{x_0}\sin\theta_0 & 0 \\ V_{x_0}\sin\theta_0 & 0 & -V_{x_0} \\ 0 & V_{x_0}\cos\theta_0 & 0 \end{pmatrix} \tag{56}$$

The second type of linearization concerns the calculations of angular speeds p, q, and r in the aircraft system of coordinates from the Euler angles and their derivatives in the inertial system of coordinates. The angular speeds p, q, and r are expressed[4] as functions of Euler angles ϕ, θ, and ψ, respectively, and their time derivatives $\dot{\phi}, \dot{\theta}$ and $\dot{\psi}$. The small perturbations theory is applied to the Euler angles ϕ, θ, and ψ, their time derivatives $\dot{\phi}, \dot{\theta}$ and $\dot{\psi}$ and to the angular speeds p, q, and r. The initial data given by the NASA DFRC laboratories are given for the angular speeds $p_0 = q_0 = r_0 = 0$, Euler angles and their time derivatives $\phi_0 = \psi_0 = 0$, $\theta_0 = \alpha_0$, and $\dot{\phi}_0 = \dot{\theta}_0 = \dot{\psi}_0 = 0$. The products of small angle variations such as

$\Delta\phi\,\Delta\theta$ and other products of such small angles can be neglected. For small angle variations $\Delta\phi$, $\Delta\theta$, and $\Delta\psi$ (sin $\Delta\phi = \Delta\phi$, cos $\Delta\phi$ = 1, ...), we can write the following system of equations :

$$\begin{pmatrix}\Delta p\\ \Delta q\\ \Delta r\end{pmatrix}=\begin{pmatrix}\Delta\dot\phi-\Delta\dot\psi\sin\theta_0\\ \Delta\dot\theta\\ \cos\theta_0\Delta\dot\psi\end{pmatrix}=\begin{pmatrix}1 & 0 & -\sin\theta_0\\ 0 & 1 & 0\\ 0 & 0 & \cos\theta_0\end{pmatrix}\begin{pmatrix}\Delta\dot\phi\\ \Delta\dot\theta\\ \Delta\dot\psi\end{pmatrix}=R\begin{pmatrix}\Delta\dot\phi\\ \Delta\dot\theta\\ \Delta\dot\psi\end{pmatrix} \tag{57}$$

where R is the following matrix:

$$R=\begin{pmatrix}1 & 0 & -\sin\theta_0\\ 0 & 1 & 0\\ 0 & 0 & \cos\theta_0\end{pmatrix} \tag{58}$$

We next replace the vectors $(\Delta u\ \Delta v\ \Delta w)^T$ given by equations (55) and (56) and $(\Delta p\ \Delta q\ \Delta r)^{\mathrm{T}}$ given by equations (57) and (58) into the two sets of equations (53.1) and (53.2) for the variations of forces $\Delta X_a, \Delta Y_a, \Delta Z_a$ and moments ΔL_a, ΔM_a and ΔN_a in the aircraft system of coordinates, and we obtain:

$$\begin{pmatrix}\Delta X_a\\ \Delta Y_a\\ \Delta Z_a\end{pmatrix}=C11\cdot P\cdot\begin{pmatrix}\Delta V_x\\ \Delta V_y\\ \Delta V_z\end{pmatrix}+C12\cdot\begin{pmatrix}\Delta X_i\\ \Delta Y_i\\ \Delta Z_i\end{pmatrix}+C13\cdot R\cdot\begin{pmatrix}\Delta\dot\phi\\ \Delta\dot\theta\\ \Delta\dot\psi\end{pmatrix}+(C14+C11\cdot P1)\cdot\begin{pmatrix}\Delta\phi\\ \Delta\theta\\ \Delta\psi\end{pmatrix}+D1\cdot\begin{pmatrix}\Delta\delta_{long\_LEF}\\ \Delta\delta_{long\_TEF}\\ \Delta\delta_{long\_AIL}\\ \Delta\delta_{long\_HT}\\ \Delta\delta_{long\_RUD}\\ \Delta\delta_{lat\_LEF}\\ \Delta\delta_{lat\_TEF}\\ \Delta\delta_{lat\_AIL}\\ \Delta\delta_{lat\_HT}\\ \Delta\delta_{lat\_RUD}\end{pmatrix} \tag{59.1}$$

$$\begin{pmatrix}\Delta L_a\\ \Delta M_a\\ \Delta N_a\end{pmatrix}=C21\cdot P\begin{pmatrix}\Delta V_x\\ \Delta V_y\\ \Delta V_z\end{pmatrix}+C22\begin{pmatrix}\Delta x_i\\ \Delta y_i\\ \Delta z_i\end{pmatrix}+C23\cdot R\begin{pmatrix}\Delta\dot{\phi}\\ \Delta\dot{\theta}\\ \Delta\dot{\psi}\end{pmatrix}+(C24+C21\cdot P1)\begin{pmatrix}\Delta\phi\\ \Delta\theta\\ \Delta\psi\end{pmatrix}+D2\begin{pmatrix}\Delta\delta_{long\_LEF}\\ \Delta\delta_{long\_TEF}\\ \Delta\delta_{long\_AIL}\\ \Delta\delta_{long\_HT}\\ \Delta\delta_{long\_RUD}\\ \Delta\delta_{lat\_LEF}\\ \Delta\delta_{lat\_TEF}\\ \Delta\delta_{lat\_AIL}\\ \Delta\delta_{lat\_HT}\\ \Delta\delta_{lat\_RUD}\end{pmatrix} \quad (59.2)$$

The development of equations (59.1) and (59.2) can be seen in Figure 6:

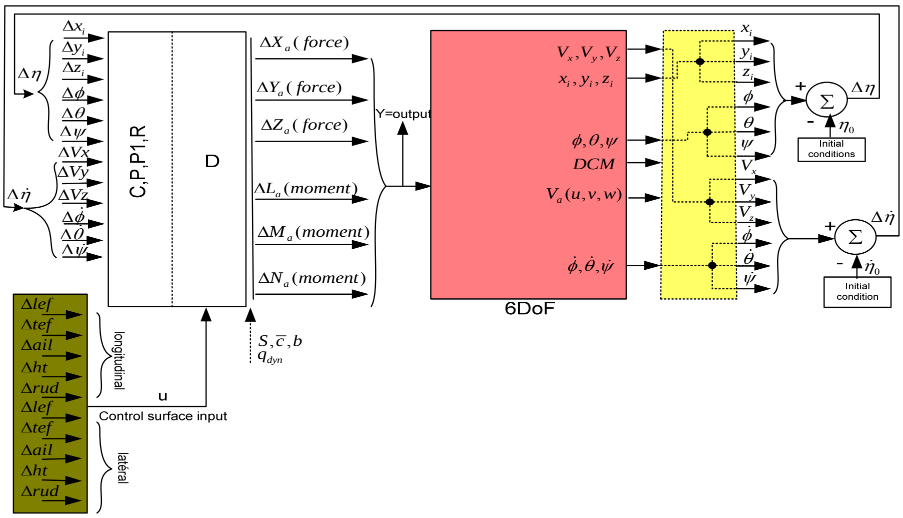

Figure 6: New scheme including the variations of state vectors and their time derivatives $\Delta\eta$ and $\Delta\dot{\eta}$ in closed loop form

In Figure 6, we introduce the following notations for the state vector variations $\Delta\eta$ and $\Delta\dot{\eta}$:

$$\Delta\eta=\begin{bmatrix}\Delta x_i & \Delta y_i & \Delta z_i & \Delta\phi & \Delta\theta & \Delta\psi\end{bmatrix}^{\mathrm{T}};\ \Delta\dot{\eta}=\begin{bmatrix}\Delta V_x & \Delta V_y & \Delta V_z & \Delta\dot{\phi} & \Delta\dot{\theta} & \Delta\dot{\psi}\end{bmatrix}^{\mathrm{T}} \quad (60)$$

2.3 AERODYNAMIC FORCES FOR RIGID-TO-RIGID AND RIGID-CONTROL INTERACTIONS MODE CALCULATIONS

To continue this work, we need to calculate the aerodynamic forces and moments variations for rigid-to-rigid Q_{rr} and rigid-to-control Q_{rc} interaction modes in the inertial system *i*. Linearization of forces and Euler angles from the aircraft system of coordinates *a* into the inertial system *i* is realized by use of the small perturbations theory.

The inertial forces variations' ΔX_i, ΔY_i, and ΔZ_i are written as functions of force variations in the aircraft system of coordinates X_a, Y_a, and Z_a by use of the CDM in a form similar to the one given by equations (18). The small perturbation theories are applied to the Euler angles, and then we obtain two sets of equations for the forces and for the moments, under the following form:

$$\begin{pmatrix} \Delta X_i \\ \Delta Y_i \\ \Delta Z_i \end{pmatrix} = F \begin{pmatrix} \Delta X_a \\ \Delta Y_a \\ \Delta Z_a \end{pmatrix} + F1 \begin{pmatrix} \Delta\phi \\ \Delta\theta \\ \Delta\psi \end{pmatrix} = F \begin{pmatrix} \Delta X_a \\ \Delta Y_a \\ \Delta Z_a \end{pmatrix} + 0 = F \begin{pmatrix} \Delta X_a \\ \Delta Y_a \\ \Delta Z_a \end{pmatrix} \tag{61.1}$$

and for the *moments*:

$$\begin{pmatrix} \Delta L_i \\ \Delta M_i \\ \Delta N_i \end{pmatrix} = F \begin{pmatrix} \Delta L_a \\ \Delta M_a \\ \Delta N_a \end{pmatrix} + F1 \begin{pmatrix} \Delta\phi \\ \Delta\theta \\ \Delta\psi \end{pmatrix} = F \begin{pmatrix} \Delta L_a \\ \Delta M_a \\ \Delta N_a \end{pmatrix} + 0 = F \begin{pmatrix} \Delta L_a \\ \Delta M_a \\ \Delta N_a \end{pmatrix} \tag{61.2}$$

where
$$F = \begin{pmatrix} \cos\theta_0 & 0 & \sin\theta_0 \\ 0 & 1 & 0 \\ -\sin\theta_0 & 0 & \cos\theta_0 \end{pmatrix} \tag{62}$$

The forces and moments variations in the aircraft system of coordinates given by equations (59.1) and (59.2) are replaced in equations (61.1) and (61.2), in which a vector of zeros of dimensions (3x10) is added, and therefore, the forces and moments are obtained in the inertial system of coordinates *i*:

$$\begin{pmatrix} \Delta X_i \\ \Delta Y_i \\ \Delta Z_i \end{pmatrix} = F \cdot C12 \cdot \begin{pmatrix} \Delta x_i \\ \Delta y_i \\ \Delta z_i \end{pmatrix} + F(C14 + C11 \cdot P1) \cdot \begin{pmatrix} \Delta\phi \\ \Delta\theta \\ \Delta\psi \end{pmatrix} + F \cdot D1 \cdot \begin{pmatrix} \Delta\delta_{long\_LEF} \\ \Delta\delta_{long\_TEF} \\ \Delta\delta_{long\_AIL} \\ \Delta\delta_{long\_HT} \\ \Delta\delta_{long\_RUD} \\ \Delta\delta_{lat\_LEF} \\ \Delta\delta_{lat\_TEF} \\ \Delta\delta_{lat\_AIL} \\ \Delta\delta_{lat\_HT} \\ \Delta\delta_{lat\_RUD} \end{pmatrix} + \tag{63.1}$$

$$+ F \cdot C11 \cdot P \cdot \begin{pmatrix} \Delta V_x \\ \Delta V_y \\ \Delta V_z \end{pmatrix} + F \cdot C13 \cdot R \cdot \begin{pmatrix} \Delta\dot\phi \\ \Delta\dot\theta \\ \Delta\dot\psi \end{pmatrix} + Zeros_{(3x10)} \cdot (0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)^T$$

$$\begin{pmatrix} \Delta L_i \\ \Delta M_i \\ \Delta N_i \end{pmatrix} = F \cdot C22 \begin{pmatrix} \Delta x_i \\ \Delta y_i \\ \Delta z_i \end{pmatrix} + F(C24 + C21 \cdot P1) \begin{pmatrix} \Delta \phi \\ \Delta \theta \\ \Delta \psi \end{pmatrix} + F \cdot D2 \begin{pmatrix} \Delta\delta_{long\_LEF} \\ \Delta\delta_{long\_TEF} \\ \Delta\delta_{long\_AIL} \\ \Delta\delta_{long\_HT} \\ \Delta\delta_{long\_RUD} \\ \Delta\delta_{lat\_LEF} \\ \Delta\delta_{lat\_TEF} \\ \Delta\delta_{lat\_AIL} \\ \Delta\delta_{lat\_HT} \\ \Delta\delta_{lat\_RUD} \end{pmatrix} + \tag{63.2}$$

$$+F \cdot C21 \cdot P \cdot \begin{pmatrix} \Delta V_x \\ \Delta V_y \\ \Delta V_z \end{pmatrix} + F \cdot C23 \cdot R \cdot \begin{pmatrix} \Delta \dot{\phi} \\ \Delta \dot{\theta} \\ \Delta \dot{\psi} \end{pmatrix} + Zeros_{(3x10)} \cdot (0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)^T$$

We arrange equations (63.1) and (63.2) in order to obtain the generalized coordinate vectors' variations $\Delta\eta$ and $\Delta\dot{\eta}$ given by equations (60), and the control vectors u and $\dot{u}$.

$$\begin{pmatrix} \Delta X_i \\ \Delta Y_i \\ \Delta Z_i \\ \Delta L_i \\ \Delta M_i \\ \Delta N_i \end{pmatrix} = \begin{bmatrix} FC12 & F(C14 + C11P1) & FD1 \\ FC22 & F(C24 + C21P1) & FD2 \end{bmatrix} \begin{pmatrix} \Delta x_i \\ \Delta y_i \\ \Delta z_i \\ \Delta \phi \\ \Delta \theta \\ \Delta \psi \\ \Delta \delta_{long\_LEF} \\ \Delta \delta_{long\_TEF} \\ \Delta \delta_{long\_AIL} \\ \Delta \delta_{long\_HT} \\ \Delta \delta_{long\_RUD} \\ \Delta \delta_{lat\_LEF} \\ \Delta \delta_{lat\_TEF} \\ \Delta \delta_{lat\_AIL} \\ \Delta \delta_{lat\_HT} \\ \Delta \delta_{lat\_RUD} \end{pmatrix} + \begin{bmatrix} FC11P & FC13R & 0 \\ FC21P & FC23R & 0 \end{bmatrix} \begin{pmatrix} \Delta V_x \\ \Delta V_y \\ \Delta V_z \\ \Delta \dot{\phi} \\ \Delta \dot{\theta} \\ \Delta \dot{\psi} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{64}$$

The system of equations (64) is simulated with the scheme shown in Figure 7.

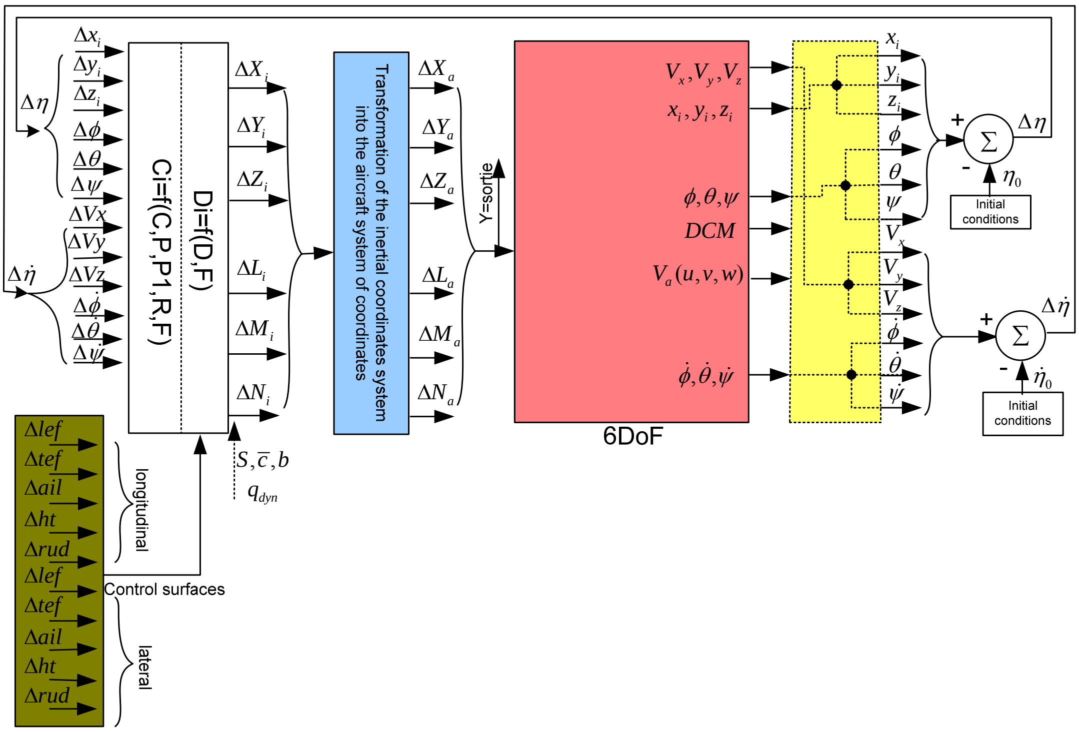

Figure 7: Scheme with force and moment variations in the aircraft system of coordinates from the inertial system of coordinates

In order to calculate the aerodynamic forces Q_{rr} and Q_{rc}, we need to write the equation of motion for the flexible aircraft structure in terms of generalized coordinates, in the following form:

$$\mathrm{M}\ddot{\eta}+\mathrm{D}\dot{\eta}+\mathrm{K}\eta + q_{dyn}\mathrm{Q}\eta = 0 \qquad (65)$$

which can be written in a simplified form as:

$$\mathrm{M}\ \ddot{\eta}+\mathrm{D}\ \dot{\eta}+\mathrm{K}\ \eta + y_1 = 0 \qquad (66)$$

where the last term of equation (66) is:

$$y_1 = F_{aero} = q_{dyn}\mathrm{Q}\eta = \left[\Delta X_i \quad \Delta Y_i \quad \Delta Z_i \quad \Delta L_i \quad \Delta M_i \quad \Delta N_i\right]^{\mathrm{T}} \qquad (67)$$

The aerodynamic forces Q have real and imaginary parts, and for this reason equation (67) can also be expressed as:

$$y_1 = q_{dyn}(Q_R + \vec{\mathrm{j}}Q_I)\eta = q_{dyn}Q_R\eta + q_{dyn}\frac{\vec{\mathrm{j}}\omega\eta}{\omega}Q_I \tag{68}$$

From the generalized coordinates definition $\eta = Ae^{j\omega t}$, where A is the amplitude of motion and ω is the oscillations frequency, we calculate the generalized coordinates derivative with time $\dot{\eta} = \vec{\mathrm{j}}\omega Ae^{j\omega t} = \vec{\mathrm{j}}\omega\eta$. The reduced frequency is given by the following equation:

$$k = \frac{\omega b}{V} = \frac{\omega\bar{c}}{2V} \tag{69}$$

We replace ω calculated with equation (69) into equation (68) and we obtain:

$$y_1 = q_{dyn}Q_R\eta + q_{dyn}\frac{\dot{\eta}}{\omega}Q_I = q_{dyn}Q_R\eta + q_{dyn}\frac{\bar{c}}{2kV}Q_I\dot{\eta} \tag{70}$$

The real parts of aerodynamic forces Q_R correspond to the state vectors x and control vectors u and the imaginary parts of aerodynamic forces Q_I correspond to the time derivatives of state vectors x and control vectors u, and therefore we can write:

$$y_1 = q_{dyn}Q_{rr}^R\eta + q_{dyn}Q_{rc}^R u + q_{dyn}\frac{\bar{c}}{2Vk}Q_{rr}^I\dot{\eta} + q_{dyn}\frac{\bar{c}}{2Vk}Q_{rc}^I\dot{u} \tag{71}$$

Equation (71) may be expressed in the form of the second state space equation:

$$y_1 = q_{dyn}\begin{pmatrix} Q_{rr}^R & \frac{\bar{c}}{2Vk}Q_{rr}^I \end{pmatrix}\begin{pmatrix} \eta \\ \dot{\eta} \end{pmatrix} + q_{dyn}\begin{pmatrix} Q_{rc}^R & \frac{\bar{c}}{2Vk}Q_{rc}^I \end{pmatrix}\begin{pmatrix} u \\ \dot{u} \end{pmatrix} \tag{72}$$

Equation (64) may be written by use of state vectors η and their time derivatives $\dot{\eta}$ and with control vectors u and their time derivatives $\dot{u}$, as follows:

$$y_1 = \begin{pmatrix} \Delta X_i \\ \Delta Y_i \\ \Delta Z_i \\ \Delta L_i \\ \Delta M_i \\ \Delta N_i \end{pmatrix} = \begin{bmatrix} F\cdot C12 & F\cdot(C14 + C11\cdot P1) & F\cdot D1 \\ F\cdot C22 & F\cdot(C24 + C21\cdot P1) & F\cdot D2 \end{bmatrix}\begin{pmatrix} \eta \\ u \end{pmatrix} + \begin{pmatrix} F\cdot C11\cdot P & F\cdot C13\cdot R & 0 \\ F\cdot C21\cdot P & F\cdot C23\cdot R & 0 \end{pmatrix}\begin{pmatrix} \dot{\eta} \\ \dot{u} \end{pmatrix} \tag{73}$$

Equation (72) can also be presented in the following form:

$$y_1 = \begin{bmatrix} \Delta X_i & \Delta Y_i & \Delta Z_i & \Delta L_i & \Delta M_i & \Delta N_i \end{bmatrix}^T = \begin{pmatrix} q_{dyn} Q_{rr}^R & q_{dyn} Q_{rc}^R \end{pmatrix} \begin{pmatrix} \eta \\ u \end{pmatrix} + \begin{pmatrix} q_{dyn} \frac{\bar{c}}{2Vk} Q_{rr}^I & q_{dyn} \frac{\bar{c}}{2Vk} Q_{rc}^I \end{pmatrix} \begin{pmatrix} \dot{\eta} \\ \dot{u} \end{pmatrix} \quad (74)$$

The Q_{rr} and Q_{rc} matrices are represented under analytical form (with 6 rows and 16 columns). By identification of the aerodynamic forces matrices given in equations (73) and (74), we calculate the terms of the real aerodynamic forces for rigid-to-rigid mode interactions Q_{rr}^R and the terms of the real aerodynamic forces for rigid-to-control mode interactions Q_{rc}^R, and thus we obtain:

$$Q_{rr}^R = \begin{bmatrix} F \cdot C12 & F \cdot (C14 + C11 \cdot P1) \\ F \cdot C22 & F \cdot (C24 + C21 \cdot P1) \end{bmatrix} = \begin{pmatrix} Q_{rr\_11}^R & Q_{rr\_12}^R \\ Q_{rr\_21}^R & Q_{rr\_22}^R \end{pmatrix} \quad (75.1)$$

$$Q_{rc}^R = \begin{pmatrix} F \cdot D1 \\ F \cdot D2 \end{pmatrix} = \begin{pmatrix} Q_{rc\_11}^R \\ Q_{rc\_21}^R \end{pmatrix} \quad (75.2)$$

The elements $Q_{rr\_11}^R, Q_{rr\_12}^R, Q_{rr\_21}^R, Q_{rr\_22}^R, Q_{rc\_11}^R, Q_{rc\_21}^R$ of real aerodynamic forces have the following analytical forms:

$$Q_{rr\_11}^R = \begin{pmatrix} qr_{1,1} & qr_{1,2} & qr_{1,3} \\ qr_{2,1} & qr_{2,2} & qr_{2,3} \\ qr_{3,1} & qr_{3,2} & qr_{3,3} \end{pmatrix}; \; Q_{rr\_12}^R = \begin{pmatrix} qr_{1,4} & qr_{1,5} & qr_{1,6} \\ qr_{2,4} & qr_{2,5} & qr_{2,6} \\ qr_{3,4} & qr_{3,5} & qr_{3,6} \end{pmatrix} \quad (76.1)$$

$$Q_{rr\_21}^R = \begin{pmatrix} qr_{4,1} & qr_{4,2} & qr_{4,3} \\ qr_{5,1} & qr_{5,2} & qr_{5,3} \\ qr_{6,1} & qr_{6,2} & qr_{6,3} \end{pmatrix}; Q_{rr\_22}^R = \begin{pmatrix} qr_{4,4} & qr_{4,5} & qr_{4,6} \\ qr_{5,4} & qr_{5,5} & qr_{5,6} \\ qr_{6,4} & qr_{6,5} & qr_{6,6} \end{pmatrix} \quad (76.2)$$

$$Q_{rc\_11}^R = \begin{pmatrix} qr_{1,7} & qr_{1,8} & qr_{1,9} & qr_{1,10} & qr_{1,11} & qr_{1,12} & qr_{1,13} & qr_{1,14} & qr_{1,15} & qr_{1,16} \\ qr_{2,7} & qr_{2,8} & qr_{2,9} & qr_{2,10} & qr_{2,11} & qr_{2,12} & qr_{2,13} & qr_{2,14} & qr_{2,15} & qr_{2,16} \\ qr_{3,7} & qr_{3,8} & qr_{3,9} & qr_{3,10} & qr_{3,11} & qr_{3,12} & qr_{3,13} & qr_{3,14} & qr_{3,15} & qr_{3,16} \end{pmatrix} \quad (76.3)$$

$$Q_{rc\_21}^{R}=\begin{pmatrix} qr_{4,7} & qr_{4,8} & qr_{4,9} & qr_{4,10} & qr_{4,11} & qr_{4,12} & qr_{4,13} & qr_{4,14} & qr_{4,15} & qr_{4,16} \\ qr_{5,7} & qr_{5,8} & qr_{5,9} & qr_{5,10} & qr_{5,11} & qr_{5,12} & qr_{5,13} & qr_{5,14} & qr_{5,15} & qr_{5,16} \\ qr_{6,7} & qr_{6,8} & qr_{6,9} & qr_{6,10} & qr_{6,11} & qr_{6,12} & qr_{6,13} & qr_{6,14} & qr_{6,15} & qr_{6,16} \end{pmatrix} \tag{76.4}$$

By identification of the imaginary parts of aerodynamic forces Q_{rr}^{I} and Q_{rc}^{I} in equations (73) and (74), we can write:

$$Q_{rr}^{I}=\begin{pmatrix} F\cdot C11\cdot P & F\cdot C13\cdot R \\ F\cdot C21\cdot P & F\cdot C23\cdot R \end{pmatrix}=\begin{pmatrix} Q_{rr\_11}^{I} & Q_{rr\_12}^{I} \\ Q_{rr\_21}^{I} & Q_{rr\_22}^{I} \end{pmatrix};\ Q_{rc}^{I}=\begin{bmatrix} 0 & 0 \end{bmatrix}^{\mathrm{T}} \tag{77}$$

The elements of the imaginary aerodynamic forces $Q_{rr\_11}^{I}, Q_{rr\_12}^{I}, Q_{rr\_21}^{I}, Q_{rr\_22}^{I}, Q_{rc\_11}^{I}$ and $Q_{rc\_21}^{I}$ have the analytical forms given in the following equations:

$$Q_{rr\_11}^{I}=\begin{pmatrix} qi_{1,1} & qi_{1,2} & qi_{1,3} \\ qi_{2,1} & qi_{2,2} & qi_{2,3} \\ qi_{3,1} & qi_{3,2} & qi_{3,3} \end{pmatrix};\quad Q_{rr\_12}^{I}=\begin{pmatrix} qi_{1,4} & qi_{1,5} & qi_{1,6} \\ qi_{2,4} & qi_{2,5} & qi_{2,6} \\ qi_{3,4} & qi_{3,5} & qi_{3,6} \end{pmatrix} \tag{78.1}$$

$$Q_{rr\_21}^{I}=\begin{pmatrix} qi_{4,1} & qi_{4,2} & qi_{4,3} \\ qi_{5,1} & qi_{5,2} & qi_{5,3} \\ qi_{6,1} & qi_{6,2} & qi_{6,3} \end{pmatrix};\quad Q_{rr\_22}^{I}=\begin{pmatrix} qi_{4,4} & qi_{4,5} & qi_{4,6} \\ qi_{5,4} & qi_{5,5} & qi_{5,6} \\ qi_{6,4} & qi_{6,5} & qi_{6,6} \end{pmatrix} \tag{78.2}$$

We next show the calculation of one single term corresponding to the real parts of the aerodynamic forces, since the same theory is used to calculate all of the terms of the real and imaginary aerodynamic forces. Please note that the *F* matrix is given by equations (62) and the C12 matrix is analytically given.

$$Q_{rr\_11}^{R}=F\cdot C12=\begin{pmatrix} \cos\theta_0 & 0 & \sin\theta_0 \\ 0 & 1 & 0 \\ -\sin\theta_0 & 0 & \cos\theta_0 \end{pmatrix}\begin{pmatrix} c_{1,4} & c_{1,5} & c_{1,6} \\ c_{2,4} & c_{2,5} & c_{2,6} \\ c_{3,4} & c_{3,5} & c_{3,6} \end{pmatrix}=\begin{pmatrix} qr_{1,1} & qr_{1,2} & qr_{1,3} \\ qr_{2,1} & qr_{2,2} & qr_{2,3} \\ qr_{3,1} & qr_{3,2} & qr_{3,3} \end{pmatrix} \tag{79}$$

The coefficients equal to zero are coloured in blue (the first two columns of C12 are zero) while the coefficients not equal to zero are coloured in red. By identification of the $Q_{rr\_11}^{R}$ matrix elements expressed by equation (79) and of the *C* matrix values given by equations (50) and other equations (not shown, due to the pages number limitation):

$$
\begin{aligned}
&qr_{1,1}=0; qr_{1,2}=0; qr_{1,3}=c_{1,6}\cdot\cos\theta_0+c_{3,6}\cdot\sin\theta_0=\bar{S}\left[-a_{4,5}\cdot\cos(\theta_0-\alpha_0)-a_{5,5}\cdot\sin(\theta_0-\alpha_0)\right]\\
&qr_{2,1}=0; qr_{2,2}=0; qr_{2,3}=c_{2,6}=\bar{S}\cdot a_{6,5}\\
&qr_{3,1}=0; qr_{3,2}=0; qr_{3,3}=-c_{1,6}\cdot\sin\theta_0+c_{3,6}\cdot\cos\theta_0=\bar{S}\left[a_{4,5}\cdot\sin(\theta_0-\alpha_0)-a_{5,5}\cdot\cos(\theta_0-\alpha_0)\right]
\end{aligned}
\tag{80}
$$

The a_i's coefficients terms in equations (80) contain stability and control derivatives as seen in equations (1)-(6). Therefore, we obtain, for all rigid aerodynamic elements Q^R, the following table, in which the first 3 elements on column 4 were given in equation (80).

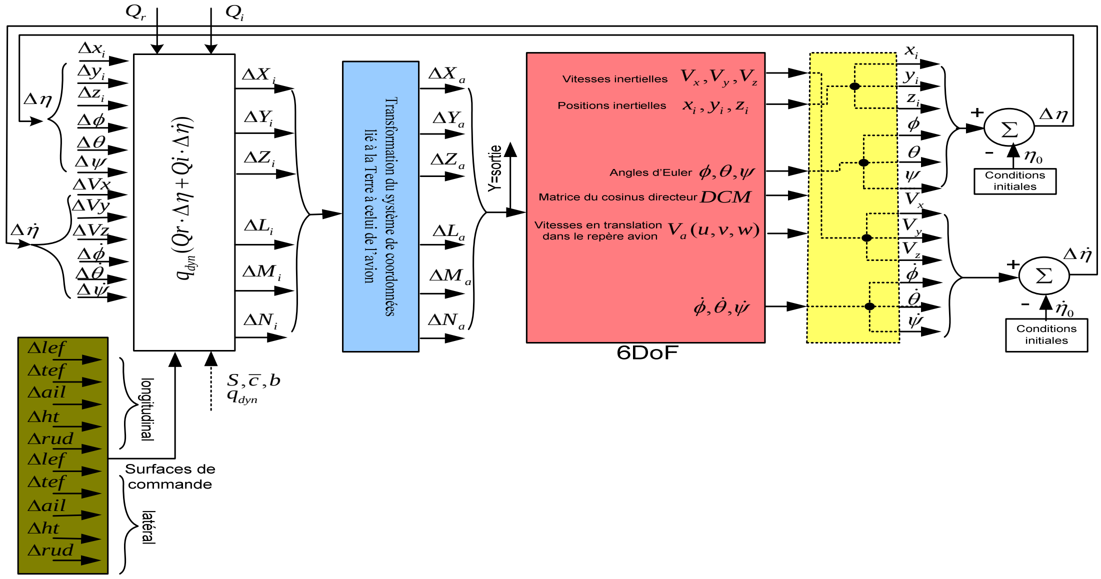

Figure 8: Simulation scheme for aerodynamic force calculations from generalized coordinates

	x_i	y_i	z_i
X	0	0	$\bar{S}\cdot\left(-C_{d_h}\cdot\cos(\theta_0-\alpha_0)-C_{lft_h}\cdot\sin(\theta_0-\alpha_0)\right)$
Y	0	0	$\bar{S}\cdot C_{y_h}$
Z	0	0	$\bar{S}\cdot\left(C_{d_h}\cdot\sin(\theta_0-\alpha_0)-C_{lft_h}\cdot\cos(\theta_0-\alpha_0)\right)$
L	0	0	$\bar{S}\cdot b\cdot\left(C_{l_h}\cdot\cos(\theta_0-\alpha_0)+C_{n_h}\cdot\sin(\theta_0-\alpha_0)\right)$
M	0	0	$\bar{S}\cdot\bar{c}\cdot C_{m_h}$
N	0	0	$\bar{S}\cdot b\cdot\left(-C_{l_h}\cdot\sin(\theta_0-\alpha_0)+C_{n_h}\cdot\cos(\theta_0-\alpha_0)\right)$

	ϕ	θ	ψ	δ
X	0	$-\bar{S}\cdot C_{d_\alpha}$	0	$-\bar{S}\cdot C_{d_\delta}$
Y	$\bar{S}\cdot C_{y_\beta}\cdot\sin\theta_0$	0	$-\bar{S}\cdot C_{y_\beta}$	$\bar{S}\cdot C_{y_\delta}$
Z	0	$-\bar{S}\cdot\left(-C_{lft_\theta}-C_{lft_\alpha}\right)$	0	$-\bar{S}\cdot C_{lft_\delta}$
L	$\bar{S}\cdot b\cdot C_{l_\beta}\cdot\sin\theta_0$	0	$-\bar{S}\cdot b\cdot C_{l_\beta}$	$\bar{S}\cdot b\cdot C_{l_\delta}$
M	0	$\bar{S}\cdot\bar{c}\cdot(C_{m_\theta}+C_{m_\alpha})$	0	$\bar{S}\cdot\bar{c}\cdot C_{m_\delta}$
N	$\bar{S}\cdot b\cdot C_{n_\beta}\cdot\sin\theta_0$	0	$-\bar{S}\cdot b\cdot C_{n_\beta}$	$\bar{S}\cdot b\cdot C_{n_\delta}$

Table 1 Real aerodynamic forces Q^R

	$\dot{x}_i$	$\dot{y}_i$	$\dot{z}_i$
X	$-\bar{S}\cdot C_{d_V}$	0	$-\bar{S}\cdot\frac{C_{d_\alpha}}{V_{x0}}$
Y	0	$\bar{S}\cdot\frac{C_{y_\beta}}{V_{x0}}$	0
Z	$-\bar{S}\cdot C_{lft_V}$	0	$-\bar{S}\cdot\frac{C_{lft_\alpha}}{V_{x0}}$
L	0	$\frac{\bar{S}\cdot b\cdot C_{l_\beta}}{V_{x0}}$	0
M	$\bar{S}\cdot\bar{c}\cdot C_{m_V}$	0	$\frac{\bar{S}\cdot\bar{c}\cdot C_{m_\alpha}}{V_{x0}}$
N	0	$\frac{\bar{S}\cdot b\cdot C_{n_\beta}}{V_{x0}}$	0

	$\dot{\phi}$	$\dot{\theta}$	$\dot{\psi}$	δ
X	$-\bar{S}\cdot C_{d_p}$	$-\bar{S}\cdot C_{d_q}$	$\bar{S}\cdot\sin\theta_0\cdot C_{d_p}$	0
Y	$\bar{S}\cdot C_{y_p}$	0	$\bar{S}\cdot\left(-C_{y_p}\cdot\sin\theta_0+C_{y_r}\cdot\cos\theta_0\right)$	0
Z	$-\bar{S}\cdot C_{lft_p}$	$-\bar{S}\cdot C_{lft_q}$	$\bar{S}\cdot\sin\theta_0\cdot C_{lft_p}$	0
L	$\bar{S}\cdot b\cdot C_{l_p}$	0	$\bar{S}\cdot b\cdot\left(-C_{l_p}\cdot\sin\theta_0+C_{l_r}\cdot\cos\theta_0\right)$	0
M	$\bar{S}\cdot\bar{c}\cdot C_{m_p}$	$\bar{S}\cdot\bar{c}\cdot C_{m_q}$	$-\bar{S}\cdot\bar{c}\cdot C_{m_p}\cdot\sin\theta_0$	0
N	$\bar{S}\cdot b\cdot C_{n_p}$	0	$\bar{S}\cdot b\cdot\left(-C_{n_p}\cdot\sin\theta_0+C_{n_r}\cdot\cos\theta_0\right)$	0

Table 2 Imaginary aerodynamic forces Q^I

The first simulation scheme (Figure 1) represents a system that uses the stability and control derivatives in the wind system of coordinates and in which the forces and moments are calculated in the aircraft system of coordinates. The second simulation scheme (Figure 8) represents an equivalent scheme similar to the first, in which the states in the inertial system of coordinates are used in the closed loop.

2.4 NUMERICAL LINEARIZATION SCHEME

The formulations already developed in Sections 1-3 will be validated for particular cases where the same types of stability and control derivatives are given in the wind system of coordinates. A problem appears if we add, subtract or change a derivative or its initial value in the formulations, because then all the formulations change. Therefore, we need to develop automatic simulation formulations. We develop a Matlab algorithm in which all of the above changes in coordinates and linearizations are automatically realized. We use the 'dlinmod' Matlab function which gives, in the state space form, the linearized form of a system built in Simulink, around a specified trim point condition. The simulation scheme presented in Figure 9 is equivalent to the scheme presented in Figure 1 and takes into consideration the stability derivatives given in the wind coordinate system.

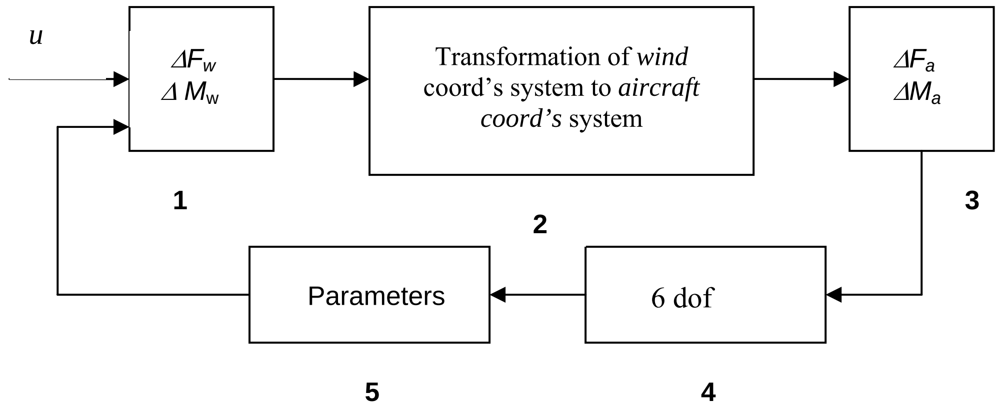

Figure 9: Simulation aircraft scheme with stability and control derivatives given in the wind coordinates system

In Block 1 of Figure 9, the forces and moments F_w and M_w are calculated from the stability and control derivatives given in the wind system of coordinates. To simulate aircraft behavior, we convert these forces and moments, determined in block 1, to forces and moments in the aircraft coordinate system shown in block 3, using block 2 for the transformation from the wind coordinate system to the aircraft coordinate system. The aircraft linear and angular speeds are the block 4 outputs and are calculated from the forces and moments in the aircraft coordinate system by use of the 6 degrees-of-freedom equations of motion. Parameters specific to the aircraft in the wind system of coordinates, such as the angle of attack, sideslip angle, and true airspeed are further calculated in block 5 and used as inputs to block 1 in the aircraft time simulation.

The scheme presented in Figure 9 around the trim point specified in the simulation is then linearized by use of the 'dlinmod' command in Matlab, where the linear relationship between the inputs u and the outputs y is obtained under state space form, in which the state vectors are $x = (u, v, w, p, q, r, \phi, \theta, \psi, x_i, y_i, z_i)$. The components of the state vectors x are the linear speeds u, v, and w, the rates p, q, and r, the angles ϕ, θ and ψ and the three positions x_i, y_i and z_i. The scheme presented in Figure 9 can further be also represented under state space form in Figure 10.

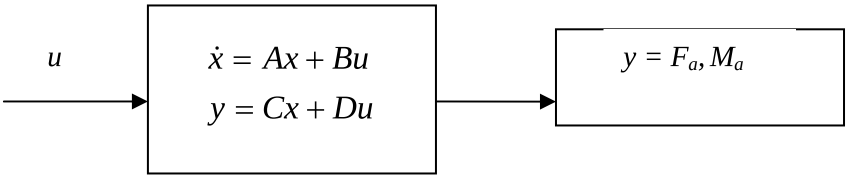

Figure 10: Equivalence with the scheme shown in Figure 9

Then, the state variables x are calculated with the first state space system equations and are further used in the second equation of the state space system to calculate the outputs y. In this case, we find the next scheme, equivalent to that shown in Figure 10, in which the block 4 outputs, from the 6 degrees of freedom (dof) equations of motion (see Figure 9), are used.
As the schemes shown in Figures 9 and 11 are compared, it is obvious that blocks 1, 2, and 4 are contained in the C and D matrices. The aircraft model thus obtained gives the variations of forces and moments calculated in the aircraft coordinate system dependent on the inputs u and state vectors x. The matrices Q_{rr} and Q_{rc}, corresponding to rigid-to-rigid and to rigid-to-control mode interactions, respectively, are determined for aerodynamic forces in this way. The states multiplying these matrices, and the forces and moments, are calculated in the inertial system of coordinates. The scheme shown in Figure 11 is redesigned by adding a first block which changes the outputs y_i into y and a second block which changes the states x into the states x_i. This new scheme is presented in Figure 12.

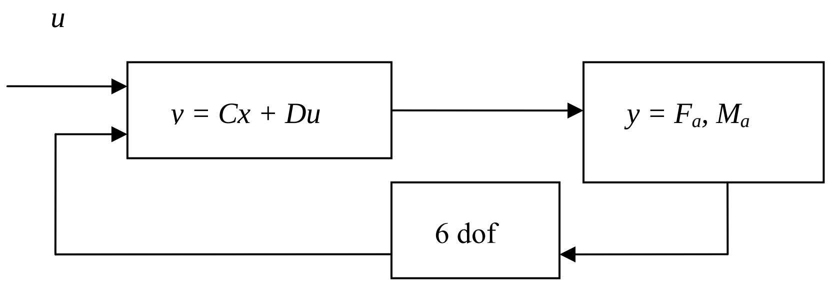

Figure 11: Equivalent schemes with the Figure 2

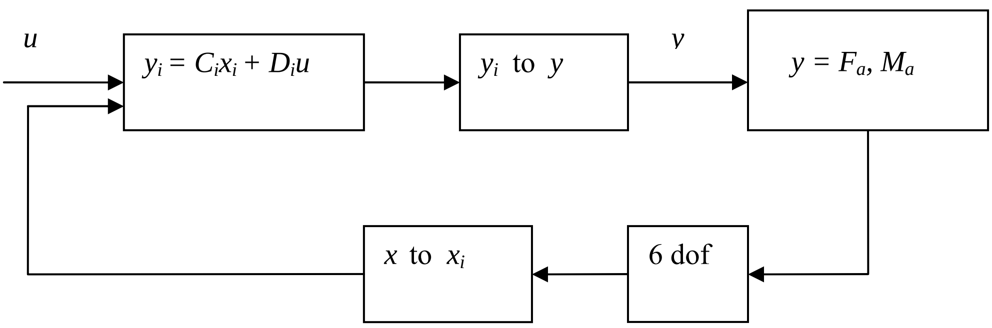

Figure 12: Simulation scheme with forces and moments calculated in the inertial system of coordinates

The changes to the coordinates implemented in the added blocks depend on the trigonometric functions of Euler angles, more specifically the inputs are related to the outputs by nonlinear functions. The two blocks are then further linearized, which requires that the Matlab command '*dlinmod*' be applied to the two blocks. For the block 'x to x_i', a linear relationship is obtained:

$$x_i = D_1 x \tag{81}$$

which represents a simplified form of a state space system where matrices A, B, and D_1 are zeros, as this block has no states. The linearization of block 'y_i to y' gives:

$$y = D_2 y_i \tag{82}$$

By using the blocks shown in Figure 12, and equations (81) and (82), we write:

$$y = D_2 y_i = D_2\left(C_i x_i + D_i u\right) = D_2\left(C_i D_1 x + D_i u\right) = D_2 C_i D_1 x + D_2 D_i u \tag{83}$$

Identifying the matrices given by equation (83) with those given by the state space equations:

$$C = D_2 C_i D_1; \qquad D = D_2 D_i \tag{84}$$

The C and D matrices are obtained from the linearization presented in Figure 9, and the D_1 and D_2 matrices are obtained by the linearization of two blocks shown in Figure 12. We calculate the C_i and D_i matrices with equations (84):

$$C_i = D_2^{-1} C D_1^{-1}; \qquad D_i = D_2^{-1} D \tag{85}$$

and the state vector x_i is:

$$x_i = \left(x_i, y_i, z_i, \phi, \theta, \psi, \dot{x}_i, \dot{y}_i, \dot{z}_i, \dot{\phi}, \dot{\theta}, \dot{\psi}\right)^T = \left[\eta_r \quad \dot{\eta}_r\right]^{\mathrm{T}} \tag{86}$$

We can write the aerodynamic force equations for the rigid modes in the following form, similar to equation (71):

$$q_{dyn}\left[Q_{rr}^R \eta_r + \frac{\bar{c}}{2Vk} Q_{rr}^I \dot{\eta}_r\right] = \left(q_{dyn} Q_{rr}^R \quad q_{dyn}\frac{\bar{c}}{2Vk} Q_{rr}^I\right) x_i = C_i x_i \tag{87}$$

The real and imaginary parts of the aerodynamic forces corresponding to the rigid modes are obtained by identification from equation (87):

$$Q_{rr}^R = \frac{1}{q_{dyn}} C_i\left(1:6, 1:6\right); \qquad Q_{rr}^I = \frac{2Vk}{\bar{c} q_{dyn}} C_i\left(1:6, 7:12\right) \tag{88}$$

The control vector u is $u = \left[\eta_c \quad 0\right]^{\mathrm{T}}$. The real and imaginary parts of the aerodynamic forces corresponding to the interactions of rigid modes with the control modes are determined with the following equations:

$$Q_{rc}^{R} = \frac{1}{q_{dyn}} D_i; \qquad Q_{rc}^{I} = 0 \tag{89}$$

The algorithm is presented in the following three steps: 1. We apply the Matlab command '*dlinmod*' to each of the 3 blocks in order to obtain C, D, D_1 and D_2 matrices; 2. Equations (88) and (89) are used to calculate the matrices Q_{rr} and Q_{rc} and 3. The initial matrices calculated with the Doublet Lattice method, DLM, or with the Constant Pressure Method, CPM, calculated by finite element software, are replaced with the matrices obtained with our algorithm. For validation purposes, the algorithm represents the numerical implementation (shown in Section 4) of analytical implementation presented in Sections 1-3. A comparison between the obtained values with the analytical formulation and the numerical formulation presented here is given in the following tables.

	x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$
x	0	0	0	0	-614.96	0
y	0	0	0	-81.33	0	468.36
z	0	0	0	0	-1502.2	0
$\dot{x}$	0	0	0	-158.72	0	914.06
$\dot{y}$	0	0	0	0	-731.29	0
$\dot{z}$	0	0	0	367.85	0	-2118.3

Table 3 Real part of Q_{rr} matrix obtained by analytical linearization

	x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$
x	0	0	-7.45E-26	0	-614.96	0
y	0	0	0	-81.33	0	468.36
z	0	0	-4.38E-25	0	-1502.2	0
$\dot{x}$	0	0	0	-158.72	0	914.06
$\dot{y}$	0	0	-6.16E-23	0	-731.29	0
$\dot{z}$	0	0	0	367.85	0	-2118.3

Table 4 Real part of Q_{rr} matrix obtained by numerical linearization

	x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$
x	-0.04	0	-0.65947	0.08	-8.6	-0.013892
y	0	-0.50226	0	2.16	0	99.091
z	-0.08	0	-1.6109	0.08	-800.16	-0.013892
$\dot{x}$	0	-0.98022	0	-5412.5	0	3948.3
$\dot{y}$	-0.9216	0	-0.78373	1.3824	-15977	-0.24005
$\dot{z}$	0	2.2717	0	-704.62	0	-2809.5

Table 5 Imaginary part of Q_{rr} matrix obtained by analytical linearization

	x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$
x	-0.04	0	-0.65947	0.08	-8.6	-0.013892
y	0	-0.50226	0	2.16	0	99.091
z	-0.08	0	-1.6109	0.08	-800.16	-0.013892
$\dot{x}$	0	-0.98022	0	-5412.5	0	3948.3
$\dot{y}$	-0.9216	0	-0.78373	1.3824	-15977	-0.24005
$\dot{z}$	0	2.2717	0	-704.62	0	-2809.5

Table 6 Imaginary part of Q_{rr} matrix obtained by numerical linearization

We can see that the obtained results are the same. Developing the numerical algorithm allowed us to verify the partial and final results for 90 flight test conditions[6].

CONCLUSIONS

In this paper, we used two approaches: analytical (Sections 1 to 3) and numerical (Section 4) to validate the aerodynamic force formulations corresponding to rigid-to-rigid and rigid-to-control interaction modes for aeroservoelasticity studies -- only from knowing the stability and control derivatives in the wind system of coordinates. These derivatives are dependent on flight regime conditions: Mach number, altitude and angle of attack. In fact, the aerodynamic forces corresponding to rigid and control interaction modes calculated with finite element software are replaced with aerodynamic forces calculated using both formulations presented here.

With another aircraft, and thus with a different set of stability and control derivatives, we will use the numerical approach combined with the theoretical approach to validate the new formulations. The numerical approach will likely be much faster than the theoretical development, because successive linearizations may take quite a long time. The theoretical approach may become much more useful in the future. The analytical approach developed in Sections 1-3 allows us to obtain the analytical formulas for all the stability and control derivatives in the inertial system from those calculated in the wind system of coordinates. Linearizations at the trim condition are performed at each calculation step.

The second approach consists of a numerical linearization of the simulation scheme in the wind reference system of coordinates. Values of stability and control derivatives obtained with this method and those calculated analytically with the first method are the same, therefore, we conclude that the expressions found for the aerodynamic forces corresponding to rigid and control modes are validated.

A comparison was done between the flutter frequencies and damping values for aeroelasticity studies (where only the elastic-elastic aerodynamic forces Q_{ee} were considered) with the flutter frequencies and damping values for aeroservoelasticity studies (where all the aerodynamic force matrix was considered). Thus, the common flutter frequencies and damping values were obtained for both aeroelasticity and aeroservoelasticity studies. Additional flutter modes were obtained for the aeroservoelasticity matrix where rigid and control modes dynamics were introduced. This comparison was another way to validate our formulation, but was not presented here in details.

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